EXERCISES 8.3, page 471

1. $\mu = (1)(0.4) + (2)(0.3) + 3(0.2) + (4)(0.1) = 2.$
   $\text{Var}(X) = (0.4)(1 - 2)^2 + (0.3)(2 - 2)^2 + (0.2)(3 - 2)^2 + (0.1)(4 - 2)^2$
   $= 0.4 + 0 + 0.2 + 0.4 = 1$
   $\sigma = \sqrt{1} = 1.$

2. $\mu = (-4)(0.1) + (-2)(0.2) + 0(0.3) + (2)(0.1) + (4)(0.3) = 0.6.$
   $\text{Var}(X) = (0.1)(-4 - 0.6)^2 + (0.2)(-2 - 0.6)^2 + (0.3)(0 - 0.6)^2$
   $+ (0.1)(2 - 0.6)^2 + (0.3)(4 - 0.6)^2$
   $= 7.24$
   $\sigma = \sqrt{7.24} \approx 2.69.$

3. $\mu = -2(\tfrac{1}{16}) + -1(\tfrac{4}{16}) + 0(\tfrac{6}{16}) + 1(\tfrac{4}{16}) + 2(\tfrac{1}{16}) = \tfrac{0}{16} = 0.$
   $\text{Var}(X) = \tfrac{1}{16}(-2-0)^2 + \tfrac{4}{16}(-1-0)^2 + \tfrac{6}{16}(0-0)^2 + \tfrac{4}{16}(1-0)^2 + \tfrac{1}{16}(2-0)^2$
   $= 1$
   $\sigma = \sqrt{1} = 1.$

4. $\mu = 10(\tfrac{1}{8}) + 11(\tfrac{2}{8}) + 12(\tfrac{1}{8}) + 13(\tfrac{2}{8}) + 14(\tfrac{1}{8}) + 15(\tfrac{1}{8})$
   $= \dfrac{99}{8} = 12.375$
   $\text{Var}(X) = \tfrac{1}{8}(10-12.375)^2 + \tfrac{2}{8}(11-12.375)^2 + \tfrac{1}{8}(12-12.375)^2 + \tfrac{2}{8}(13-12.375)^2$
   $+ \tfrac{1}{8}(14-12.375)^2 + \tfrac{1}{8}(15-12.375)^2$
   $= 0.7051 + 0.4727 + 0.0176 + 0.0977 + 0.3301 + 0.8613 = 2.4845.$
   $\sigma = \sqrt{2.4845} = 1.58$

5. $\mu = 0.1(430) + (0.2)(480) + (0.4)(520) + (0.2)(565) + (0.1)(580)$
   $= 518.$
   $\text{Var}(X) = 0.1(430 - 518)^2 + (0.2)(480 - 518)^2 + (0.4)(520 - 518)^2$
   $+ (0.2)(565 - 518)^2 + (0.1)(580 - 518)^2$
   $= 1891.$
   $\sigma = \sqrt{1891} \approx 43.49.$

6. $\mu = (0.15)(-198) + (0.30)(-195) + (0.10)(-193) + (0.25)(-188)$
   $+ (0.20)(-185)$

$= -191.5.$

$\text{Var}(X) = (0.15)(-198 + 191.5)^2 + (0.30)(-195 + 191.5)^2$
$+ (0.25)(-188 + 191.5)^2 + (0.20)(-185 + 191.5)^2$
$= 21.525.$

$\sigma = \sqrt{21.525} \approx 4.64.$

7. The mean of the histogram in Figure (b) is more concentrated about its mean than the histogram in Figure (a). Therefore, the histogram in Figure (a) has the larger variance.

8. a.

9. $E(X) = 1(0.1) + 2(0.2) + 3(0.3) + 4(0.2) + 5(0.2) = 3.2.$
$\text{Var}(X) = (0.1)(1 - 3.2)^2 + (0.2)(2 - 3.2)^2 + (0.3)(3 - 3.2)^2$
$+ (0.2)(4 - 3.2)^2 + (0.2)(5 - 3.2)^2$
$= 1.56$

10. $E(X) = 0.05(1) + (0.1)(2) + (0.15)(3) + (0.2)(4) + (0.15)(5)$
$+ (0.1)(6) + (0.15)(7) + (0.1)(8)$
$= 4.7$

$\text{Var}(X) = (0.05)(1 - 4.7)^2 + (0.1)(2 - 4.7)^2 + (0.15)(3 - 4.7)^2$
$+ (0.2)(4 - 4.7)^2 + (0.15)(5 - 4.7)^2 + (0.1)(6 - 4.7)^2$
$+ (0.15)(7 - 4.7)^2 + (0.1)(8 - 4.7)^2$
$= 4.01.$

11. $\mu = \dfrac{1+2+3+\cdots+8}{8} = 4.5$

$V(X) = \tfrac{1}{8}(1-4.5)^2 + \tfrac{1}{8}(2-4.5)^2 + \cdots + \tfrac{1}{8}(8-4.5)^2 = 5.25$

12. a. X gives the minimum age requirement for a regular driver's license.

b.

x	15	16	17	18	19	21
P(X = x)	0.02	0.30	0.08	0.56	0.02	0.02

c. $\mu = E(X)$

$= (0.02)(15) + (0.3)(16) + (0.08)(17) + (0.56)(18) + (0.02)(19)$
$+ (0.02)(21)$
$= 17.34.$

$V(X) = (0.02)(15 - 17.34)^2 + (0.3)(16 - 17.34)^2 + (0.08)(17 - 17.34)^2$
$+ (0.56)(18 - 17.34)^2 + (0.02)(19 - 17.34)^2 + (0.02)(21 - 17.34)^2$
$= 1.2244$

$\sigma = \sqrt{1.2244} \approx 1.11.$

13. a. Let X be the annual birth rate during the years 1981 - 1990.

b.

x	15.5	15.6	15.7	15.9	16.2	16.7
$P(X = x)$	0.2	0.1	0.3	0.2	0.1	0.1

c. $E(X) = (0.2)(15.5) + (0.1)(15.6) + (0.3)(15.7) + (0.2)(15.9)$
$+ (0.1)(16.2) + (0.1)(16.7)$
$= 15.84.$

$V(X) = (0.2)(15.5 - 15.84)^2 + (0.1)(15.6 - 15.84)^2$
$+ (0.3)(15.7 - 15.84)^2 + (0.2)(15.9 - 15.84)^2$
$+ (0.1)(16.2 - 15.84)^2 + (0.1)(16.7 - 15.84)^2$
$= 0.1224$

$\sigma = \sqrt{0.1224} \approx 0.350.$

14. a. Venture A

$\mu = (-20{,}000)(0.3) + (40{,}000)(0.4) + (50{,}000)(0.3) = 25{,}000.$

$V(X) = (0.3)(-20{,}000 - 25{,}000)^2 + (0.4)(40{,}000 - 25{,}000)^2$
$+ (0.3)(50{,}000 - 25{,}000)^2$
$= 8.85 \times 10^8.$

Venture B

$\mu = (-15{,}000)(0.2) + (30{,}000)(0.5) + (40{,}000)(0.3) = 24{,}000.$

$V(X) = (0.2)(-15{,}000 - 24{,}000)^2 + (0.5)(30{,}000 - 24{,}000)^2$
$+ (0.3)(40{,}000 - 24{,}000)^2$
$= 3.99 \text{ x } 10^8.$

b. Venture *A*   c. Venture *B*

15. a. Mutual Fund *A*

$\mu = (0.2)(-4) + (0.5)(8) + (0.3)(10) = 6.2$, or \$620.

$V(X) = (0.2)(-4 - 6.2)^2 + (0.5)(8 - 6.2)^2 + (0.3)(10 - 6.2)^2$
$= 26.76$, or \$267,600.

Mutual Fund B

$\mu = (0.2)(-2) + (0.4)(6) + (0.4)(8)$
$= 5.2$, or \$520.

$V(X) = (0.2)(-2 - 5.2)^2 + (0.4)(6 - 5.2)^2 + (0.4)(8 - 5.2)^2$
$= 13.76$, or \$137,600.

b. Mutual Fund A c. Mutual Fund B

16. $\mu = (0.01)(0) + (0.03)(1) + (0.05)(2) + (0.11)(3) + (0.13)(4) + (0.24)(5) + (0.22)(6) + (0.16)(7) + (0.05)(8)$
$= 5.02$

$V(X) = (0.01)(0 - 5.02)^2 + (0.03)(1 - 5.02)^2 + (0.05)(2 - 5.02)^2 + (0.11)(3 - 5.02)^2 + (0.13)(4 - 5.02)^2 + (0.24)(5 - 5.02)^2 + (0.22)(6 - 5.02)^2 + (0.16)(7 - 5.02)^2 + (0.05)(8 - 5.02)^2$
$= 3.0596$.

17. $\text{Var}(X) = (0.4)(1)^2 + (0.3)(2)^2 + (0.2)(3)^2 + (0.1)(4)^2 - (2)^2 = 1$.

18. $\text{Var}(X) = (0.01)(0)^2 + (0.03)(1)^2 + (0.05)(2)^2 + (0.11)(3)^2 + (0.13)(4)^2 + (0.24)(5)^2 + (0.22)(6)^2 + (0.16)(7)^2 + (0.05)(8)^2 - (5.02)^2$
$= 3.0596$.

19. $\mu = [\frac{10}{500}(180) + \frac{20}{500}(190) + \cdots + \frac{5}{500}(350)] = 239.6$, or \$239,600.

$V(X) = [\frac{10}{500}(180 - 239.6)^2 + \frac{20}{500}(190 - 239.66)^2 + \cdots + \frac{5}{500}(350 - 239.66)^2][(1000)^2]$
$= 1443.84 \times 10^6$ dollars.

$\sigma = \sqrt{1443.84 \times 10^6} = 37.998 \times 10^3$, or \$37,998.

20. The mean is given by

$$\frac{(94.5)(4) + (84.5)(8) + (74.5)(12) + (64.5)(4) + (54.5)(2)}{30} \approx 77.167,$$

or approximately 77.2.

$$\text{Var}(x) = \left(\tfrac{4}{30}\right)(94.5 - 77.167)^2 + \left(\tfrac{8}{30}\right)(84.5 - 77.167)^2 + \left(\tfrac{12}{30}\right)(74.5 - 77.167)^2 + \left(\tfrac{4}{30}\right)(64.5 - 77.167)^2 + \left(\tfrac{2}{30}\right)(54.5 - 77.167)^2 \approx 112.88889$$

Therefore, $\sigma = \sqrt{112.8889} \approx 10.62$.

21. The mean is given by

$$22\left(\tfrac{1332}{27127}\right) + 27\left(\tfrac{4219}{27127}\right) + 32\left(\tfrac{6345}{27127}\right) + 37\left(\tfrac{7598}{27127}\right) + 42\left(\tfrac{7633}{27127}\right) \approx 34.9456, \text{ or } 34.95.$$

$$\text{Var}(x) = \left(\tfrac{1332}{27127}\right)(22 - 34.9456)^2 + \left(\tfrac{4219}{27127}\right)(27 - 34.9456)^2 + \cdots$$
$$+ \left(\tfrac{7633}{27127}\right)(42 - 34.9456)^2 \approx 35.26222.$$

Therefore, $\sigma = 5.938$, or 5.94.

22. The mean is given by

$$22\left(\tfrac{6178}{14807}\right) + 27\left(\tfrac{3689}{14807}\right) + 32\left(\tfrac{2219}{14807}\right) + 37\left(\tfrac{1626}{14807}\right) + 42\left(\tfrac{1095}{14807}\right) \approx 27.8705, \text{ or } 27.87.$$

$$\text{Var}(x) = \left(\tfrac{6178}{14807}\right)(22 - 27.8075)^2 + \left(\tfrac{3689}{14807}\right)(27 - 27.8075)^2 + \cdots$$
$$+ \left(\tfrac{1095}{14807}\right)(42 - 27.8075)^2 \approx 41.0439$$

Therefore, $\sigma = 6.4066$, or approximately 6.41.

23. a. Using Chebychev's inequality we have

$P(\mu - k\sigma \leq X \leq \mu + k\sigma) \geq 1 - 1/k^2$.

$\mu - k\sigma = 42 - k(2) = 38$, and $k = 2$,

and $P(\mu - k\sigma \leq X \leq \mu + k\sigma) \geq 1 - 1/(2)^2$

$\geq 1 - 1/4$

$\geq 3/4$, or at least 0.75.

b. Using Chebychev's inequality we have

$P(\mu - k\sigma \leq X \leq \mu + k\sigma) \geq 1 - 1/k^2$.

$\mu - k\sigma = 42 - k(2) = 32$, and $k = 5$,

and $P(\mu - k\sigma \leq X \leq \mu + k\sigma) \geq 1 - 1/(5)^2$

$\geq 1 - 1/25 \geq 24/25$, or at least 0.96.

24. a. Using Chebychev's inequality we have

$P(\mu - k\sigma \leq X \leq \mu + k\sigma) \geq 1 - 1/k^2$.

$\mu - k\sigma = 20 - k(3) = 15$, and $k = 5/3$,

and $P(\mu - k\sigma \leq X \leq \mu + k\sigma) \geq 1 - 1/(5/3)^2$

$\geq 1 - 9/25 \geq 16/25$, or at least 0.64.

b. Using Chebychev's inequality we have

$P(\mu - k\sigma \leq X \leq \mu + k\sigma) \geq 1 - 1/k^2$.

$\mu - k\sigma = 20 - k(3) = 10$, and $k = 10/3$, and $P(\mu - k\sigma \le X \le \mu + k\sigma) \ge 1 - 1/(10/3)^2 \ge 1 - 9/100 \ge 91/100$, or at least 0.91

.

25. Here $\mu = 50$ and $\sigma = 1.4$. Now, we require that $c = k\sigma$, or $k = \dfrac{c}{1.4}$.

Next, we solve $0.96 = 1 - \left(\dfrac{1.4}{c}\right)^2$; $\dfrac{1.96}{c^2} = 0.04$; $c^2 = \dfrac{1.96}{0.04} = 49$, or $c = 7$.

26. Using Chebychev's inequality we have $P(\mu - k\sigma \le X \le \mu + k\sigma) \ge 1 - 1/k^2$.
Here $k = 2$, so $P(\mu - k\sigma \le X \le \mu + k\sigma) \ge 1 - 1/2^2 = 3/4$. This means that at least 75 percent of the values are expected to lie between $\mu - 2\sigma$ and $\mu + 2\sigma$.

27. Using Chebychev's inequality we have $P(\mu - k\sigma \le X \le \mu + k\sigma) \ge 1 - 1/k^2$.
Here, $\mu - k\sigma = 24 - k(3) = 20$, and $k = 4/3$.
So $P(\mu - k\sigma \le X \le \mu + k\sigma) \ge 1 - 1/(4/3)^2 \ge 1 - 9/16 = 7/16$, or at least 0.4375.

28. Here $\mu = 200$ and $\sigma = 2$.
a. In this case k satisfies $200 - 2k = 190$ and $200 + 2k = 210$
from which we deduce that $k = \frac{10}{2} = 5$. Therefore,

$$P(190 \le X \le 210) \ge 1 - \frac{1}{5^2} = \frac{96}{100}, \text{ or } 0.96.$$

b. In this case k satisfies $200 - 2k = 180$ and $200 + 2k = 220$, from which we deduce that $k = 20/2 = 10$. Therefore, $P(180 \le X \le 220) \ge 1 - \dfrac{1}{10^2} = \dfrac{99}{100}$
or 0.01. Then the estimated number of Christmas tree lights that will burn out is given by $(0.01)(150{,}000) = 1500$.

29. Using Chebychev's inequality we have

$$P(\mu - k\sigma \le X \le \mu + k\sigma) \ge 1 - 1/k^2.$$

Here, $\mu - k\sigma = 42{,}000 - k(500) = 40{,}000$, and $k = 4$.
So $P(\mu - k\sigma \le X \le \mu + k\sigma) \ge 1 - 1/(4)^2 \ge 1 - 1/16 \ge 15/16$, or at least 0.9375.

30. Here $\mu = 5$ and $\sigma = 0.02$. Next, we require that $c = k\sigma$, or $k = \dfrac{c}{0.02}$. Next, solve

$0.96 = 1 - \left(\frac{0.02}{c}\right)^2$; $0.04 = \dfrac{0.0004}{c^2} = 0.01$; $c^2 = \dfrac{0.0004}{0.04} = 0.01$ and $c = 0.1$.

We conclude that for $P(5 - c \le X \le 5 + c) \ge 0.96$. We require that $c \ge 0.1$.

31. True. This follows from the definition.

32. True. If $k \le 1$, then $\frac{1}{k^2} \ge 1$ and so $1 - \frac{1}{k^2} \le 0$ and the inequality (9) becomes trivial.

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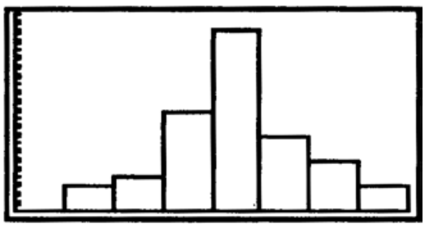
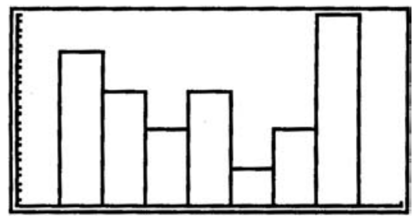
1. a.



b. $\mu = 4$ and $\sigma \approx 1.40$

2. a.

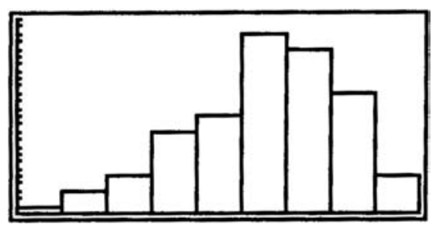


b. $\mu = 4$ and $\sigma \approx 2.28$

3. a.

b. $\mu = 17.34$ and $\sigma \approx 1.11$

4. a.



b. $\mu = 5.02$ and $\sigma \approx 1.749$

5. a. Let X denote the random variable that gives the weight of a carton of sugar.
b. The probability distribution for the random variable X is

x	4.96	4.97	4.98	4.99	5.00	5.01	5.02	5.03	5.04	5.05	5.06
P(X = x)	$\frac{3}{30}$	$\frac{4}{30}$	$\frac{4}{30}$	$\frac{1}{30}$	$\frac{1}{30}$	$\frac{5}{30}$	$\frac{3}{30}$	$\frac{3}{30}$	$\frac{4}{30}$	$\frac{1}{30}$	$\frac{1}{30}$

$\mu = 5.00467 \approx 5.00$; $V(X) = 0.0009$; $\sigma = \sqrt{0.0009} = 0.03$

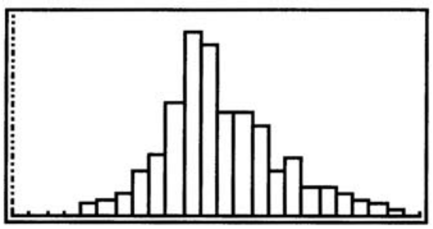
6. a. Let X be the random variable that is the score of a student in a mathematics examination.
b. The probability distribution for the random variable X is

x	58	66	68	69	70	71	72	73
P(X = x)	0.04	0.08	0.12	0.04	0.08	0.04	0.08	0.04

x	74	75	85	87	88	90	92	94	98
P(X = x)	0.08	0.04	0.08	0.04	0.04	0.08	0.04	0.04	0.04

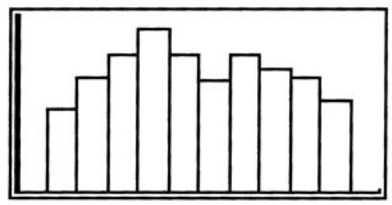
c. $\mu = 76.92$; $V(X) = 110.9536$; $\sigma \approx 10.533$.

7. a.



b. $\mu = 65.875$ and $\sigma = 1.73$.

8. a.



b. $\mu = 5.47$ that is, approximately 19.5 years old and $\sigma = 2.68$.