6.1 Exercises

(1) $\cos x \tan x = \cos x \cdot \dfrac{\sin x}{\cos x} = \boxed{\sin x}$

(2) $\sec^2 x (1 - \sin^2 x) = \dfrac{1}{\cos^2 x} \cdot \cos^2 x = \boxed{1}$

(3) $\dfrac{\sec \theta}{\csc \theta} = \sec \theta \cdot \dfrac{1}{\csc \theta} = \dfrac{1}{\cos \theta} \cdot \sin \theta = \boxed{\tan \theta}$

(4) $\dfrac{\tan^2 x}{\sec^2 x} = \dfrac{\sin^2 x}{\cos^2 x} \cdot \dfrac{1}{\sec^2 x} = \dfrac{\sin^2 x}{\cos^2 x} \cdot \cos^2 x = \boxed{\sin^2 x}$

(5) $\cot\left(\frac{\pi}{2} - x\right) \cos x = \tan x \cdot \cos x = \dfrac{\sin x}{\cos x} \cdot \cos x = \boxed{\sin x}$

(6) $\dfrac{1}{\cot^2 x + 1} = \dfrac{1}{\csc^2 x} = \boxed{\sin^2 x}$

(7) $\sec^2 x \tan^2 x + \sec^2 x = \sec^2 x (\tan^2 x + 1)$

$= \sec^2 x (\sec^2 x)$

$= \boxed{\sec^4 x}$

(8) $\dfrac{\csc^2 x - 1}{\csc x - 1} = \dfrac{(\cancel{\csc x - 1})(\csc x + 1)}{\cancel{\csc x - 1}} = \boxed{\csc x + 1}$

(9) $1 - 2\sin^2 x + \sin^4 x$

$(1 - \sin^2 x)(1 - \sin^2 x)$

$(\cos^2 x)(\cos^2 x)$

$\boxed{\cos^4 x}$

(10) $\sec^4 x - \tan^4 x$

$(\sec^2 x - \tan^2 x)(\sec^2 x + \tan^2 x)$

$(1)(\sec^2 x + \tan^2 x) = 1 + \tan^2 x + \tan^2 x$

$\boxed{1 + 2\tan^2 x}$

(11) $\sec^3 x - \sec^2 x - \sec x + 1$

$\sec^2 x\,(\sec x - 1) - 1\,(\sec x - 1)$

$(\sec^2 x - 1)(\sec x - 1)$

$\boxed{\tan^2 x\,(\sec x - 1)}$

(12) $(\sin x + \cos x)^2$

$\sin^2 x + 2\sin x \cos x + \cos^2 x$

$\sin^2 x + \cos^2 x + 2\sin x \cos x$

$\boxed{1 + 2\sin x \cos x}$

(13) $(\tan x + \sec x)(\tan x - \sec x) = \tan^2 x - \sec^2 x$

$= \boxed{-1}$

(14) $(\csc x + 1)(\csc x - 1)$

$\csc^2 x - 1$

$\boxed{\cot^2 x}$

(15) $(5 - 5\sin x)(5 + 5\sin x)$

$25 - 25\sin^2 x$

$25(1 - \sin^2 x)$

$25(\cos^2 x)$

$\boxed{25\cos^2 x}$

* (16) $\tan x + \dfrac{\cos x}{1+\sin x}$

Change denominator first!! and write $\tan x$ as $\dfrac{\sin x}{\cos x}$

$$\frac{\sin x}{\cos x} + \frac{\cos x}{1+\sin x}\cdot\frac{(1-\sin x)}{(1-\sin x)}$$

$$\frac{\sin x}{\cos x} + \frac{\cos x\,(1-\sin x)}{1-\sin^2 x} = \frac{\sin x}{\cos x} + \frac{\cos x\,(1-\sin x)}{\cos^2 x}$$

or $= \dfrac{\sin x}{\cos x} + \dfrac{1-\sin x}{\cos x}$

$$= \frac{\sin x\cos x}{\cos^2 x} + \frac{\cos x - \sin x\cos x}{\cos^2 x}$$

$$= \frac{\cos x}{\cos^2 x} = \frac{1}{\cos x} = \boxed{\sec x}$$

*(17) Typo. I am so sorry. I made a typo.

This one does not simplify prettily.

It was supposed to be $\frac{\cos x}{1+\sin x} + \frac{1+\sin x}{\cos x}$

(18) $\frac{5}{\tan x + \sec x} \cdot \frac{(\tan x - \sec x)}{(\tan x - \sec x)}$

$= \frac{5(\tan x - \sec x)}{\tan^2 - \sec^2 x} = \frac{5(\tan x - \sec x)}{-1(\sec^2 x - \tan^2 x)}$

$= \boxed{-5(\tan x - \sec x)}$

(19) $\frac{\tan^2 x}{\csc x + 1} \cdot \frac{(\csc x - 1)}{(\csc x - 1)}$

$= \frac{\tan^2 x(\csc x - 1)}{\csc^2 x - 1} = \frac{\tan^2 x(\csc x - 1)}{\cot^2 x}$

$= \frac{\frac{\sin^2 x}{\cos^2 x}(\csc x - 1)}{\frac{\cos^2 x}{\sin^2 x}} = \frac{\sin^2 x}{\cos^2 x} \cdot \frac{\sin^2 x}{\cos^2 x}(\csc x - 1)$

$= \boxed{\tan^4 x(\csc x - 1)}$