6.1 Using Fundamental Identities

Lesson objectives: I can use trig identities to

- ❑ Simplify trig expressions
- ❑ Rewrite trig expressions

Fundamental Trigonometric Identities

Reciprocal Identities

$$\sin u = \frac{1}{\csc u} \qquad \cos u = \frac{1}{\sec u} \qquad \tan u = \frac{1}{\cot u}$$

$$\csc u = \frac{1}{\sin u} \qquad \sec u = \frac{1}{\cos u} \qquad \cot u = \frac{1}{\tan u}$$

Quotient Identities

$$\tan u = \frac{\sin u}{\cos u} \qquad \cot u = \frac{\cos u}{\sin u}$$

Pythagorean Identities

$$\sin^2 u + \cos^2 u = 1 \qquad 1 + \tan^2 u = \sec^2 u \qquad 1 + \cot^2 u = \csc^2 u$$

Cofunction Identities

$$\sin\left(\frac{\pi}{2} - u\right) = \cos u \qquad \cos\left(\frac{\pi}{2} - u\right) = \sin u$$

$$\tan\left(\frac{\pi}{2} - u\right) = \cot u \qquad \cot\left(\frac{\pi}{2} - u\right) = \tan u$$

$$\sec\left(\frac{\pi}{2} - u\right) = \csc u \qquad \csc\left(\frac{\pi}{2} - u\right) = \sec u$$

Even/Odd Identities

$$\sin(-u) = -\sin u \qquad \cos(-u) = \cos u \qquad \tan(-u) = -\tan u$$

$$\csc(-u) = -\csc u \qquad \sec(-u) = \sec u \qquad \cot(-u) = -\cot u$$

Simplifying Trig expressions

- Seems to mean to write it in terms of a single trig function without fractions.
- Sometimes it can only be simplified into terms of two trig functions.

Strategies

- Lots of substitution
- Using different variations of the Pythagorean identities

  Ex: $\sin^2 x + \cos^2 x = 1$ also means...

  $\sin^2 x = 1 - \cos^2 x$

  $\cos^2 x = 1 - \sin^2 x$

$(\tan\theta)^2 = \left(\frac{\sin\theta}{\cos\theta}\right)^2$

$\tan^2\theta = \frac{\sin^2\theta}{\cos^2\theta}$

- Factoring
- Multiplying by different forms of 1

  $\frac{\sin x}{\sin x}$ or $\frac{(1-\cos x)}{(1-\cos x)}$

Simplify each expression.

1. cot x sin x

$\frac{\cos x}{\sin x} \cdot \sin x = \boxed{\cos x}$

2. $\sin u(\csc u - \sin u)$

$\sin u\left(\frac{1}{\sin u} - \sin u\right)$

$1 - \sin^2 u$

$\boxed{\cos^2 u}$

3. $\frac{\csc x}{\cot x}$

$$\csc x \cdot \frac{1}{\cot x} = \frac{1}{\sin x} \cdot \frac{\sin x}{\cos x} = \frac{1}{\cos x} = \boxed{\sec x}$$

4. $\sec\beta \cdot \dfrac{\sin\beta}{\tan\beta}$ $= \dfrac{1}{\cancel{\cos\beta}} \cdot \dfrac{\cancel{\sin\beta}}{1} \cdot \dfrac{\cancel{\cos\beta}}{\cancel{\sin\beta}} = \boxed{1}$

5. $\sin\left(\frac{\pi}{2} - x\right)\csc x$ $= \cos x \ \csc x$

$= \dfrac{\cos x}{1} \cdot \dfrac{1}{\sin x} = \dfrac{\cos x}{\sin x} = \boxed{\cot x}$

6. $\frac{cos^2 y}{1 - \sin y} = \frac{1-\sin^2 y}{1-\sin y} = \frac{(\cancel{1-\sin y})(1+\sin y)}{\cancel{1-\sin y}} = \boxed{1+\sin y}$

Factor and use fundamental identities to simplify.

7. sin x cos² x – sin x

$\sin x\,(\cos^2 x - 1)$

$-\sin x\,(1-\cos^2 x)$

$-\sin x\,(\sin^2 x)$

$\boxed{-\sin^3 x}$

8 ~~9~~. $\frac{cos^2 x - 4}{\cos x - 2}$

$$\frac{(\cos x + 2)\cancel{(\cos x - 2)}}{\cancel{\cos x - 2}} = \boxed{\cos x + 2}$$

9.

~~10~~. $\tan^4 x + 2\tan^2 x + 1$

$$(\tan^2 x + 1)(\tan^2 x + 1)$$

$$\sec^2 x \cdot \sec^2 x$$

$$\boxed{\sec^4 x}$$

10. ~~12~~ ~~11~~. $\underbrace{\csc^3 x - \csc^2 x} - \underbrace{\csc x + 1}$

$\csc^2 x(\csc x - 1) - 1(\csc x - 1)$

$(\csc^2 x - 1)(\csc x - 1)$

$\boxed{\cot^2 x(\csc x - 1)}$

Add or subtract as indicated and use fundamental identities to simplify.

11. ~~13~~ ~~12~~. $\frac{1}{1+\cos x} + \frac{1}{1-\cos x}$

$$\frac{1(1-\cos x) + 1(1+\cos x)}{(1+\cos x)(1-\cos x)} = \frac{1 - \cancel{\cos x} + 1 + \cancel{\cos x}}{}$$

$$= \frac{2}{1-\cos^2 x} = \frac{2}{\sin^2 x}$$

$$= \boxed{2\csc^2 x}$$

Rewrite the expression so that it is not in fractional form.

12. ~~13. 14.~~ $\cancel{\dfrac{\sin^2 y}{1 - \cos y}}$

$$\frac{1}{1-\cos y} \cdot \frac{(1+\cos y)}{(1+\cos y)} = \frac{1+\cos y}{1-\cos^2 y}$$

$$= \frac{1+\cos y}{\sin^2 y}$$

$$= \frac{1}{\sin^2 y} + \frac{\cos y}{\sin^2 y}$$

$$= \boxed{\csc^2 y + \cot y \cdot \csc y}$$