Verifying Trigonometric Identities

Objective: To verify that two expressions are equivalent. That is, we want to verify that what we have is an identity.

- To do this, we generally pick the expression on one side of the given identity and manipulate that expression until we get the other side.
- In most cases, it is best to start with the more complex looking side and try to simply to match the less complex side.
- You must be very familiar with the fundamental trigonometric identities, especially the Pythagorean Identities. In some cases, a direct substitution using these fundamental identities will verify the identity you are trying to prove (Exercise 8 at the end of this document is one example).
- Some special approaches are useful for certain types of identities, which are provided below.

Identity Type	**Verification**	**Approach**
<u>**Type 1**</u>: Sometimes it is easier if we just rewrite everything in terms of sine and cosine to see if the expression simplifies.	Verify: $\mathbf{\cot x + 1 = \csc x\,(\cos x + \sin x)}$ $\text{RHS} \rightarrow \mathbf{\csc x\,(\cos x + \sin x)} = \frac{1}{\sin x}(\cos x + \sin x)$ $= \frac{\cos x}{\sin x} + \frac{\sin x}{\sin x}$ $= \mathbf{\cot x + 1}$	• Start with more complex RHS. • Rewrite $\csc x$ in terms of sine or cosine. • Remember, $\csc x = 1/\sin x$ • Also note, $\cos x/\sin x = \cot x$ • The RHS simplifies to original LHS.
<u>**Type 2**</u>: In some cases, the more complex side involves a fraction that can be split up. Then we rewrite everything in terms of sine and cosine.	Verify: $\mathbf{\frac{\tan t - \cot t}{\sin t \cos t} = \sec^2 t - \csc^2 t}$ $\text{LHS} \rightarrow \mathbf{\frac{\tan t - \cot t}{\sin t \cos t}} = \frac{\tan t}{\sin t \cos t} - \frac{\cot t}{\sin t \cos t}$ $= \tan t \cdot \frac{1}{\sin t \cos t} - \cot t \cdot \frac{1}{\sin t \cos t}$ $= \frac{\sin t}{\cos t} \cdot \frac{1}{\sin t \cos t} - \frac{\cos t}{\sin t} \cdot \frac{1}{\sin t \cos t}$ $= \frac{1}{\cos^2 t} - \frac{1}{\sin^2 t}$ $= \mathbf{\sec^2 t - \csc^2 t}$	• Start with the more complex LHS. • Rewrite the LHS as difference of two fractions. • Split out $\tan t$ and $\cot t$ to make it easier to simplify. • Notice in the first term, the $\sin t$ cancels out; and in the second term, $\cos t$ cancels out. • The new terms are reciprocal identities • The LHS simplifies to the original RHS.

Verifying Trigonometric Identities

Identity Type	Verification	Approach
Type 3: Using the property of conjugates is sometimes helpful. For an expression like $a+b$, the conjugate would be $a-b$. When you multiply conjugates, you often get a more useful expression, e.g., $(a+b)(a-b)$. Sometimes multiplying by the conjugate will simplify an expression and help in verifying the given identity.	Verify: $$\frac{\cos x}{1-\sin x}=\frac{1+\sin x}{\cos x}$$ $\text{RHS} \rightarrow \frac{1+\sin x}{\cos x}=\frac{1+\sin x}{\cos x}\left(\frac{1-\sin x}{1-\sin x}\right)$ $=\frac{1-\sin^2 x}{\cos x\,(1-\sin x)}$ $=\frac{\cos^2 x}{\cos x\,(1-\sin x)}$ $=\frac{\cos x\cos x}{\cos x\,(1-\sin x)}$ $=\frac{\cos x}{1-\sin x}$	• We could start with either side; but here we will start with the RHS. • The conjugate of the numerator $1+\sin x$ is $1-\sin x$. • Multiply by $\frac{1-\sin x}{1-\sin x}=1$ • Remember, $1-\sin^2 x=\cos^2 x$ • Once we reduce the fraction, we get the LHS of the original identity.
Type 4: Combining fractions before using identities may be an appropriate strategy.	Verify: $$\frac{1}{1-\sin\alpha}+\frac{1}{1+\sin\alpha}=2\sec^2\alpha$$ $\text{LHS} \rightarrow \frac{1}{1-\sin\alpha}+\frac{1}{1+\sin\alpha}=\frac{1}{1-\sin\alpha}\left(\frac{1+\sin\alpha}{1+\sin\alpha}\right)+\frac{1}{1+\sin\alpha}\left(\frac{1-\sin\alpha}{1-\sin\alpha}\right)$ $=\frac{(1+\sin\alpha)+(1-\sin\alpha)}{(1-\sin\alpha)(1+\sin\alpha)}$ $=\frac{2}{1-\sin^2\alpha}$ $=\frac{2}{\cos^2\alpha}$ $=2\sec^2\alpha$	• Notice that the denominators of the fractions on the LHS are conjugates. • So we will use the property of conjugates to combine the LHS fractions and simplify.