6.5 Double Angle Identities

4.27.16

a) Prove the double-angle sine identity, $\sin 2x = 2\sin x\cos x$.

$$\sin(2x) = \sin(x+x) = \sin x\cos x + \sin x \cdot \cos x.$$
$$= 2\sin x\cos x$$

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b) Prove the double-angle cosine identity, $\cos 2x = \cos^2 x - \sin^2 x$.

$$\cos 2x = \cos(x+x) = \cos x \cdot \cos x - \sin \cdot \sin x$$
$$= \cos^2 x - \sin^2 x$$

c) The double-angle cosine identity, $\cos 2x = \cos^2 x - \sin^2 x$, can be expressed as $\cos 2x = 1 - 2\sin^2 x$ or $\cos 2x = 2\cos^2 x - 1$. Derive each identity.

$$\cos 2x = \cos^2 x - \sin^2 x$$
$$= 1 - \sin^2 x - \sin^2 x$$
$$= 1 - 2\sin^2 x$$

$$\cos 2x = \cos^2 x - \sin^2 x$$
$$= \cos^2 x - (1 - \cos^2 x)$$
$$= \cos^2 x - 1 + \cos^2 x$$
$$= 2\cos^2 x - 1$$

d) Derive the double-angle tan identity, $\tan 2x = \dfrac{2\tan x}{1-\tan^2 x}$.

$$\tan 2x = \tan(x+x) = \frac{\sin x \cdot \cos x + \sin x\cos x}{\cos x \cdot \cos x - \sin x \sin x} = \frac{(2\sin x\cos x) \cdot \frac{1}{\cos^2 x}}{(\cos^2 x - \sin^2 x)\frac{1}{\cos^2 x}}$$

$$= \frac{\frac{2\sin\cos x}{\cos^2 x}}{\frac{\cos^2 x - \sin^2 x}{\cos^2 x}}$$

$$= \frac{2 \cdot \frac{\sin x}{\cos x}}{1 - \frac{\sin^2 x}{\cos^2 x}}$$

$$= \frac{2\tan x}{1 - \tan^2 x}$$