

6.5 Double Angle Identities

4.27.16

a) Prove the double-angle sine identity, $\sin 2x = 2 \sin x \cos x$.

$$\begin{aligned}\sin(2x) &= \sin(x+x) = \sin x \cos x + \sin x \cdot \cos x \\ &= 2 \sin x \cos x\end{aligned}$$

b) Prove the double-angle cosine identity, $\cos 2x = \cos^2 x - \sin^2 x$.

$$\begin{aligned}\cos 2x &= \cos(x+x) = \cos x \cdot \cos x - \sin x \cdot \sin x \\ &= \cos^2 x - \sin^2 x\end{aligned}$$

c) The double-angle cosine identity, $\cos 2x = \cos^2 x - \sin^2 x$, can be expressed as $\cos 2x = 1 - 2\sin^2 x$ or $\cos 2x = 2\cos^2 x - 1$. Derive each identity.

$$\begin{aligned}\cos 2x &= \cos^2 x - \sin^2 x \\ &= 1 - \sin^2 x - \sin^2 x \\ &= 1 - 2\sin^2 x\end{aligned}$$

$$\begin{aligned}\cos 2x &= \cos^2 x - \sin^2 x \\ &= \cos^2 x - (1 - \cos^2 x) \\ &= \cos^2 x - 1 + \cos^2 x \\ &= 2\cos^2 x - 1\end{aligned}$$

d) Derive the double-angle tan identity, $\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$.

$$\tan 2x = \tan(x+x) = \frac{\sin x \cdot \cos x + \sin x \cos x}{\cos x \cdot \cos x - \sin x \sin x}$$

$$\begin{aligned}&= \frac{\left(\frac{2 \sin x \cos x}{\cos^2 x} \right) \cdot \frac{1}{\cos^2 x}}{\left(\frac{\cos^2 x - \sin^2 x}{\cos^2 x} \right) \cdot \frac{1}{\cos^2 x}} \\ &= \frac{2 \frac{\sin x \cos x}{\cos^2 x}}{\frac{\cos^2 x - \sin^2 x}{\cos^2 x}}\end{aligned}$$

$$\begin{aligned}&= \frac{2 \cdot \frac{\sin x}{\cos x}}{1 - \frac{\sin^2 x}{\cos^2 x}} \\ &= \frac{2 \tan x}{1 - \tan^2 x}\end{aligned}$$