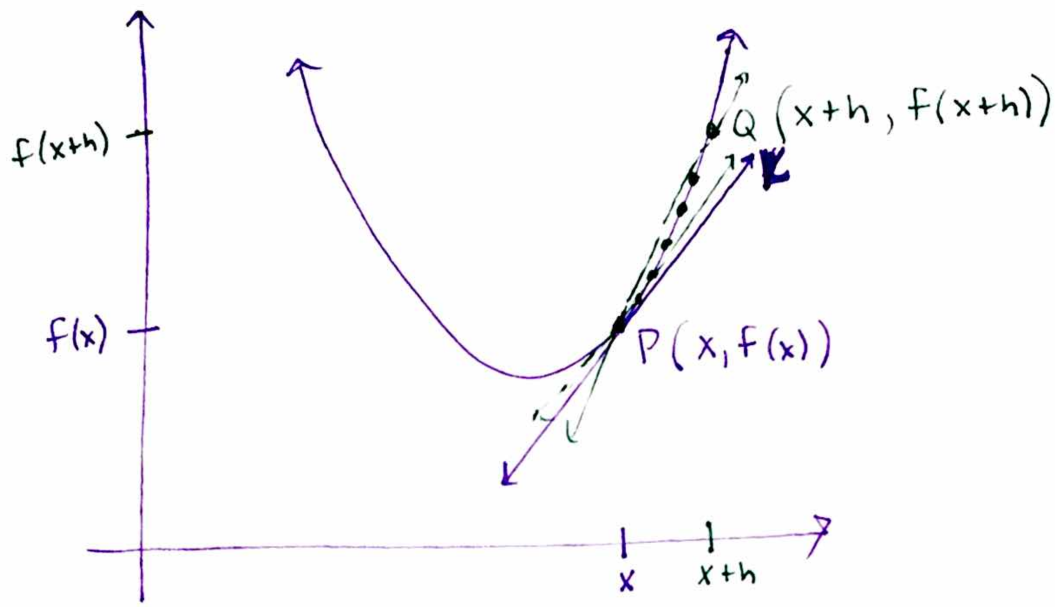
Unit 7 - Derivatives

7.1 - The Tangent Line Problem



Problem: How to find the slope of the tangent line at point P.

Answer: Approximate the slope of line k using the secant that passes through P and Q, where Q is a second point on the curve.

$$\text{slope} = m = \frac{y_2 - y_1}{x_2 - x_1} \qquad \text{slope } \overline{PQ} = \frac{f(x+h) - f(x)}{(x+h) - x}$$

$$= \frac{f(x+h) - f(x)}{h}$$

as Q gets closer to P, h gets closer to 0.

$$m_{\text{tangent}} = \boxed{\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}}$$

Ex - Find the equation of the tangent line to the graph of f at the indicated point.

① $f(x) = x^2 + 1$; $(2, 5)$

$$m = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{(x+h)^2 + 1 - (x^2 + 1)}{h}$$

$$= \lim_{h \to 0} \frac{x^2 + 2xh + h^2 \cancel{+1} \cancel{-x^2} \cancel{-1}}{h}$$

$$= \lim_{h \to 0} 2x + h$$

$m = 2x$ at $(2, 5)$

$m = 2(2) = 4$

$y = mx + b$

$5 = 4(2) + b$

$5 = 8 + b$

$-3 = b$

$$\boxed{y = 4x - 3}$$

② $f(x) = x^3 + 2x \; ; \; (1,3)$

$$m = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{(x+h)^3 + 2(x+h) - (x^3 + 2x)}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{x^3} + 3x^2h + 3xh^2 + h^3 + \cancel{2x} + 2h - \cancel{x^3} - \cancel{2x}}{h}$$

$$= \lim_{h \to 0} 3x^2 + 3xh + h^2 + 2$$

$m = 3x^2 + 2$ at $(1,3)$

$m = 3(1)^2 + 2$

$m = 5$

$y = mx + b$

$3 = 5(1) + b$

$3 = 5 + b$

$-2 = b$

$\boxed{y = 5x - 2}$

(3) $f(x) = \dfrac{1}{x+3}$ at $(1, \frac{1}{4})$

$$m = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\dfrac{1}{(x+h+3)} - \dfrac{1}{(x+3)}}{h}$$

$$= \lim_{h \to 0} \frac{1(x+3) - 1(x+h+3)}{(x+h+3)(x+3)} \cdot \frac{1}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{x}\cancel{+3} - \cancel{x} - \cancel{h} \cancel{-3}}{\cancel{h}(x+h+3)(x+3)}$$

$$= \lim_{h \to 0} \frac{-1}{(x+h+3)(x+3)}$$

$$m = \frac{-1}{(x+3)^2} \quad \text{at } (1, \tfrac{1}{4})$$

$$m = \frac{-1}{(1+3)^2} = \frac{-1}{16}$$

$$y = mx + b$$

$$\frac{1}{4} = -\frac{1}{16}(1) + b$$

$$\frac{5}{16} = b$$

$$\boxed{y = -\frac{1}{16}x + \frac{5}{16}}$$