7.2 The Definition of Derivative

$f'(x)$ = "f prime of x" or "the derivative of $f(x)$"

other notations: $\frac{dy}{dx}$, y', $\frac{d}{dx}[f(x)]$, $D_x[y]$

Definition : The derivative of f at x is

$$\boxed{f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}}$$

Examples: Find $f'(x)$

1. $f(x) = x^3 + 2x$

$$f'(x) = \lim_{h\to 0} \frac{(x+h)^3 + 2(x+h) - (x^3 + 2x)}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{x^3} + 3x^2h + 3xh^2 + h^3 \cancel{+2x} + 2h \cancel{- x^3} \cancel{- 2x}}{h}$$

$$= \lim_{h\to 0} 3x^2 + 3xh + h^2 + 2$$

$$= 3x^2 + 3(0)\cancel{x} + 0^2 + 2$$

$$\boxed{f'(x) = 3x^2 + 2}$$

② $f(x) = \sqrt{2x}$

add to 0

$$f'(x) = \lim_{h \to 0} \frac{\left(\sqrt{2(x+h)} - \sqrt{2x}\right)}{h} \cdot \frac{\left(\sqrt{2(x+h)} + \sqrt{2x}\right)}{\left(\sqrt{2(x+h)} + \sqrt{2x}\right)}$$

$$= \lim_{h \to 0} \frac{2(x+h) - 2x}{h\left(\sqrt{2(x+h)} + \sqrt{2x}\right)}$$

$$= \lim_{h \to 0} \frac{\cancel{2x} + 2h \cancel{- 2x}}{h\left(\sqrt{2(x+h)} + \sqrt{2x}\right)}$$

$$= \lim_{h \to 0} \frac{2}{\sqrt{2(x+h)} + \sqrt{2x}}$$

$$= \frac{2}{\sqrt{2x} + \sqrt{2x}}$$

$$= \frac{2}{2\sqrt{2x}} = \frac{1}{\sqrt{2x}}$$

$$\boxed{f'(x) = \frac{1}{\sqrt{2x}}}$$