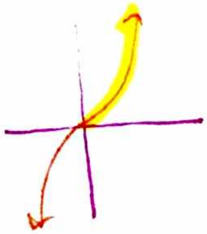
Derivatives and Rates of Change

1. Suppose $s(t) = 2t^3$ represents the position of a race car along a straight track, measured in feet from the starting line at time t seconds.
   a. What is the average rate of change of $s(t)$ from $t = 2$ to $t = 3$?



rate of change $= \frac{\Delta s}{\Delta t} = \frac{54-16}{3-2} = \boxed{38 \text{ ft/sec}}$ average velocity → speed with direction

$s(3) = 54$

$s(2) = 16$

   b. What is the instantaneous rate of change (derivative) of the same race car at time $t = 2$?
   *Remember the derivative is the slope (rate of change) of the tangent line at a point on a curve.
   *The instantaneous rate of change measures the rate of change, or slope, of a curve at a certain instant.
   Thus, the instantaneous rate of change is given by the derivative.

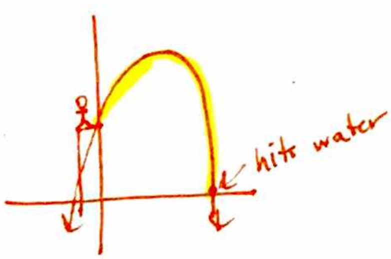
$s(t) = 2t^3$

$s'(t) = 6t^2$

$s'(2) = 6(2)^2$

$= \boxed{24 \text{ ft/sec}}$ velocity at 2 sec.

2. At time t = 0, a diver jumps from a diving board that is 32 feet above the water. The position of the diver is given by
   $s(t) = -16t^2 + 16t + 32$ where s is measured in feet and t is measured in seconds.
   a. When does the diver hit the water? when $s(t) = 0$



$0 = -16t^2 + 16t + 32$

$0 = -16(t^2 - t - 2)$

$0 = -16(t-2)(t+1)$

$t - 2 = 0$ $\quad t + 1 = 0$

$\boxed{t = 2}$ $\quad t = -1$

after $\boxed{\text{at 2 seconds}}$

   b. What is the diver's velocity at impact?

$v(t) = s'(t) = -32t + 16$

$v(2) = -32(2) + 16$

$= -64 + 16$

$= \boxed{-48 \text{ ft/sec}}$