

6.2 Exercises

$$(12) \sec y \cdot \cos y = 1$$

$$\text{LHS} \rightarrow \sec y \cdot \cos y = \frac{1}{\cos y} \cdot \cos y = 1 \rightarrow \text{RHS}$$

$$(14) \frac{\sin^2 t}{\tan^2 t} = \cos^2 t$$

$$\text{LHS} \rightarrow \sin^2 t \cdot \frac{1}{\tan^2 t} = \sin^2 t \cdot \frac{\cos^2 t}{\sin^2 t} = \cos^2 t \rightarrow \text{RHS}$$

$$(16) \cos^2 \beta - \sin^2 \beta = 2 \cos^2 \beta - 1$$

$$\begin{aligned} \text{LHS} &\rightarrow \cos^2 \beta - \sin^2 \beta \\ &= \cos^2 \beta - (1 - \cos^2 \beta) \\ &= \cos^2 \beta - 1 + \cos^2 \beta \\ &= 2 \cos^2 \beta - 1 \rightarrow \text{RHS} \end{aligned}$$

$$(18) 2 - \csc^2 z = 1 - \cot^2 z$$

$$\begin{aligned} \text{LHS} &\rightarrow 2 - \csc^2 z = 2 - (1 + \cot^2 z) \\ &= 2 - 1 - \cot^2 z \\ &= 1 - \cot^2 z \rightarrow \text{RHS} \end{aligned}$$

$$(20) \tan^2 y (\csc^2 y - 1) = 1$$

$$\text{LHS} \rightarrow \frac{\sin^2 y}{\cos^2 y} \left(\frac{1}{\sin^2 y} - 1 \right)$$

$$= \frac{\sin^2 y}{\cos^2 y} \cdot \frac{1}{\sin^2 y} - 1 \cdot \frac{\sin^2 y}{\cos^2 y}$$

$$= \frac{1}{\cos^2 y} - \frac{\sin^2 y}{\cos^2 y}$$

$$= \frac{1 - \sin^2 y}{\cos^2 y}$$

$$= \frac{\cos^2 y}{\cos^2 y} = 1 \rightarrow \text{RHS}$$

$$(34) \sec^6 x (\sec x \tan x) - \sec^4 x (\sec x \tan x) = \sec^5 x \tan^3 x$$

$$\text{LHS} \rightarrow \sec^7 x \tan x - \sec^5 x \tan x$$

$$= \sec^5 x \tan x (\sec^2 x - 1)$$

$$= \sec^5 x \tan x (\tan^2 x)$$

$$= \sec^5 x \tan^3 x \rightarrow \text{RHS}$$

$$(42) (\sec \theta - \tan \theta) (\csc \theta + 1) = \cot \theta$$

$$\text{LHS: } \sec \theta \cdot \csc \theta + \sec \theta - \tan \theta \csc \theta - \tan \theta$$

$$= \frac{1}{\cos \theta} \cdot \frac{1}{\sin \theta} + \frac{1}{\cos \theta} - \frac{\sin \theta}{\cos \theta} \cdot \frac{1}{\sin \theta} - \frac{\sin \theta}{\cos \theta}$$

$$= \frac{1}{\cos \theta \sin \theta} + \frac{1 (\sin \theta)}{\cos \theta (\sin \theta)} - \frac{\sin \theta}{\cos \theta \sin \theta} - \frac{\sin \theta \cdot \sin \theta}{\cos \theta \sin \theta}$$

$$= \frac{1 - \sin^2 \theta}{\cos \theta \sin \theta} = \frac{\cos^2 \theta}{\cos \theta \sin \theta} = \frac{\cos \theta}{\sin \theta} = \cot \theta \quad \uparrow \text{RHS}$$

$$(44) \sec^2 y - \cot^2 \left(\frac{\pi}{2} - y \right) = 1$$

$$\text{LHS: } 1 + \tan^2 y - \tan^2 y = 1 \rightarrow \text{RHS}$$

$$(46) \sec^2 \left(\frac{\pi}{2} - x \right) - 1 = \cot^2 x$$

$$\text{LHS: } \csc^2 x - 1 = \cot^2 x \rightarrow \text{RHS}$$