6.5 Exercises Name KEY

Prove each identity.

1) $\cos 2x + 2\sin^2 x = 1$

$(2\cos^2 x - 1) + 2(1-\cos^2 x)$

$\cancel{2\cos^2 x} - 1 + 2 - \cancel{2\cos^2 x}$

$-1 + 2 = \boxed{1}$ ✓

2) $\dfrac{2}{1+\cos 2x} = \sec^2 x$ $\dfrac{1}{\cos^2 x}$

$\dfrac{2}{1+(2\cos^2 x - \cancel{1})}$

$\dfrac{2}{2\cos^2 x} = \dfrac{\cancel{2}}{\cancel{2}} \cdot \dfrac{1}{\cos^2 x} =$

$= \dfrac{1}{\cos^2 x} = \boxed{\sec^2 x}$ ✓

3) $\dfrac{\sin 2x}{\cos 2x + \sin^2 x} = 2\tan x$ $\dfrac{2\cdot\sin x}{\cos x}$

$\dfrac{2\sin x \cdot \cos x}{\cos^2 x - \cancel{\sin^2 x} + \cancel{\sin^2 x}}$

$= \dfrac{2\sin x \cdot \cancel{\cos x}}{\cos^2 x}$

$= \dfrac{2\sin x}{\cos x} = \boxed{2\tan x}$ ✓

4) $\dfrac{2\sin x\cos x}{\cos^2 x - \sin^2 x} = \tan 2x$

$= \dfrac{2\tan x}{1-\tan^2 x}$

$= \dfrac{2\cdot\dfrac{\sin x}{\cos x}}{\dfrac{\cos^2 x}{\cos^2 x}\cdot\dfrac{1}{1} - \dfrac{\sin^2 x}{\cos^2 x}}$

$= \dfrac{2\dfrac{\sin x}{\cos x}}{\dfrac{\cos^2 x - \sin^2 x}{\cos^2 x}}$

$= 2\dfrac{\sin x}{\cancel{\cos x}} \cdot \dfrac{\cos^{\cancel{2}} x}{\cos^2 x - \sin^2 x}$

$= \boxed{\dfrac{2\sin x\cos x}{\cos^2 x - \sin^2 x}}$

$Ex: x^4 - y^4 = (x^2-y^2)(x^2+y^2)$

5) $\cos^4 x - \sin^4 x = \cos \underset{2\cos^2 x - 1}{2x}$

$= (\cos^2 x - \sin^2 x)(\cos^2 x + \sin^2 x)$

$= \cos^2 x - (1-\cos^2 x)(1 - \cancel{\sin^2 x} + \cancel{\sin^2 x})$

$= (\cos^2 x - 1 + \cos^2 x)(1)$

$= \boxed{2\cos^2 x - 1}$ ✓

6) $1-(\sin x + \cos x)^2 = -\sin \underset{-2\sin x \cdot \cos x}{2x}$

$1-(\sin x + \cos x)(\sin x + \cos x)$

$1-(\sin^2 x + 2\sin x \cos x + \cos^2 x)$

$1-(2\sin x \cdot \cos x + 1)$

$= \cancel{1} - 2\sin x \cdot \cos x - \cancel{1}$

$= \boxed{-2\sin x \cdot \cos x}$

7) $\dfrac{2(\tan x - \cot x)}{\tan^2 x - \cot^2 x} = \sin \underset{2\sin x \cdot \cos x}{2x}$

$= \dfrac{2\left(\dfrac{\sin x}{\cos x} - \dfrac{\cos x}{\sin x}\right)}{\dfrac{\sin^2 x}{\cos^2 x} - \dfrac{\cos^2 x}{\sin^2 x}}$

$= \dfrac{2\left(\dfrac{\sin^2 x}{\sin x \cdot \cos x} - \dfrac{\cos^2 x}{\sin x \cdot \cos x}\right)}{\dfrac{\sin^4 x}{\sin^2 x \cdot \cos^2 x} - \dfrac{\cos^4 x}{\sin^2 x \cdot \cos^2 x}}$

$= 2\dfrac{\cancel{\sin^2 x - \cos^2 x}}{\cancel{\sin x \cdot \cos x}} \cdot \dfrac{\sin^2 x \cdot \cos^2 x}{\cancel{\sin^2 x - \cos^2 x}}$

$= \boxed{2\sin x \cdot \cos x}$ ✓

8) $\left(\dfrac{1}{1-\tan x}\right)\left(\dfrac{1}{1+\tan x}\right) = \tan \underset{\frac{2\tan x}{1-\tan^2 x}}{2x}$

$\dfrac{(1+\tan x)}{(1+\tan x)} \qquad \dfrac{(1-\tan x)}{(1-\tan x)}$

$= \dfrac{1+\tan x}{1-\tan^2 x} - \dfrac{1-\tan x}{1-\tan^2 x}$

$= \dfrac{\cancel{1} + \tan x - \cancel{1} + \tan x}{1-\tan^2 x}$

$= \boxed{\dfrac{2\tan x}{1-\tan^2 x}}$ ✓

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