Investigation: Taking Apart the Area

In the previous investigation, you explored ways of rewriting quadratic expressions using an area model. The expressions were represented in two forms:

- Factored $(x+ m)(x + n)$
- Expanded $x^2 + bx + c$.

In this investigation, you will explore how to use the area model when the quadratic expressions have a coefficient of x^2 that is not 1. You will be able to rewrite these expressions in factored or expanded form.

Consider the situation where a housing developer is preparing to order mulch for landscaping. She must know the area of each lot to determine how much to order.

1. There are three lots that have exactly the same width and length. The developer finds that the total area of all three lots together can be represented by $3x^2 + 21x + 30$ where *x* is the length of the original square lot.

   a. Write an expression that represents the area of one of the three lots? Explain your reasoning.

   b. Use an area model to rewrite the expression as the product of the length and width of each of these lots.

   c. The total area of the three lots can be written with equivalent expressions. Write three different expressions that represent the total area of all three lots. Identify which one is in expanded form and which one is in *completely* factored form.

2. In the previous problem, all three coefficients were multiples of 3 making it possible to rewrite the expression to represent the area of each lot and the dimensions of each lot. Use similar reasoning to work with each of the following expressions and rewrite them into a completely factored form.

   a. $2x^2 + 24x + 70$

   b. $5x^2 + 10x - 40$

   c. $10x^2 + 40x + 30$

   d. $-3x^2 + 12x + 36$

   e. $4x^2 + 60x + 216$

   f. $-5x^2 + 15x – 10$

   g. $8x^2 – 80x + 168$

   h. $3x^2 – 3$

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3. Another lot has a total area represented by the expression $2x^2 + 5x + 3$.
   a. How does the area model to the right represent the expression for the total area of the lot?

x^2	x^2	x	x	x
x	x	1	1	1

   b. How is this different than the expressions in problems 1 and 2?

   c. Write the factored form that represents the product of the length and width of the lot.

4. Determine if the model is correct. If so, complete the model. If not, cross it out.

a. $2x^2 + 7x + 3$

	___	___	___
___	x^2	x^2	x
___	$3x$	$3x$	3

b. $2x^2 + 7x + 12$

	___	___
___	$2x^2$	x
___	$6x$	12

c. $2x^2 + 4x + 5$

	___	___
___	$2x^2$	$3x$
___	x	5

d. $3x^2 + 17x + 10$

	___	___
___	$3x^2$	$2x$
___	$15x$	10

e. $3x^2 + 4x + 6$

	___	___
___	$3x^2$	$2x$
___	$2x$	6

f. $3x^2 + 10x – 8$

	___	___
___	$3x^2$	$-2x$
___	$12x$	-8

5. Use the area model to factor each of the following.

a. $3x^2 + 4x + 1$

b. $2x^2 + 5x + 2$

c. $7x^2 + 20x – 3$

d. $5x^2 - 14x – 3$

e. $4x^2 + 13x – 12$

f. $2x^2 + 9x – 35$

The process of rewriting the expanded expression into a factored expression is called *factoring*. The area model uses the fact that a rectangle's area is the <u>product of the length and width</u> (factored form) and the <u>sum of the individual areas</u> (expanded form). It is a visual representation of this algebraic procedure and can be used for any factorable expression. *Yes, there are expressions that cannot be factored.*

6. Use the area model to rewrite each expression in factored form.
   a. $4x^2 - 32x + 60$
   b. $2x^2 + 13x + 15$
   c. $x^2 + 10x + 25$
   d. $x^2 - 25$
   e. $5x^2 - 30x + 45$

Summarize the Mathematics

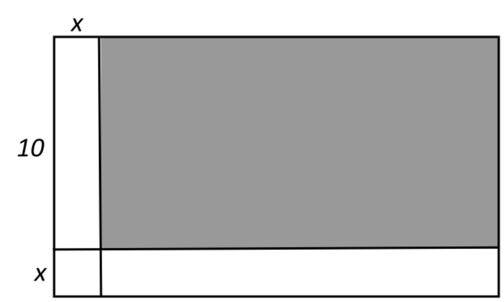
a. Create a quadratic expression and write it in both factored and expanded form.

b. How would you go about finding the factored form for the quadratic expressions like $x^2 + bx + c$?

c. How would you modify your strategy in part b for quadratic expressions of the form $ax^2 + bx + c$?

Check Your Understanding

a. The area of the rectangle shown in the figure is $3x^2 + 38x + 80$. What is the area of the shaded region?



b. Factor each of the following expressions

i. $6x^2 + 6x – 12$ ii. $x^2 – 49$ iii. $2x^2 – 5x – 3$ iv. $x^2 + 14x + 49$

v. $5x^2 - 5x – 30$ vi. $3x^2 - 7x + 2$ vii. $x^2 – 20x + 100$ viii. $4x^2 – 81$

Taking Apart the Area (Teacher Notes)

1. This task uses context to help students recognize the common factor.
   a. Each lot has an area that is ⅓ of the original area. Therefore, the area of one lot is $x^2 + 7x + 10$.
   b. $(x + 5)\ (x + 2)$

x^2	$5x$
$2x$	10

   c. Expanded form $3x^2 + 21x + 30$
   $= 3\ (x^2 + 7x + 10)$
   Completely factored $= 3(x + 5)(x + 2)$

2. Students should factor each of the expressions and write the completely factored form.

a. $2x^2 + 24x + 70$
$2(x^2 + 12x + 35)$
$2(x + 7)\ (x + 5)$

	x	5
x	x^2	$5x$
7	$7x$	35

b. $5x^2 + 10x - 40$
$5(x^2 + 2x - 8)$
$5(x + 4)(x - 2)$

	x	4
x	x^2	$4x$
-2	$-2x$	-8

c. $10x^2 + 40x + 30$
$10\ (x^2 + 4x + 3)$
$10\ (x + 1)(x + 3)$

	x	1
x	x^2	x
3	$3x$	3

d. $-3x^2 + 12x + 36$
$-3(x^2 - 4x - 12)$
$-3(x - 6)(x + 2)$

	x	-6
x	x^2	$-6x$
2	$2x$	-12

e. $4x^2 + 60x + 216$
$4(x^2 + 15x + 54)$
$4(x + 9)(x + 6)$

	x	9
x	x^2	$9x$
6	$6x$	54

f. $-5x^2 + 15x - 10$
$-5(x^2 - 3x + 2)$
$-5(x - 1)(x - 2)$

	x	-1
x	x^2	$-x$
-2	$-2x$	2

g. $8x^2 - 80x + 168$
$8(x^2 - 10x + 21)$
$8(x - 3)(x - 7)$

	x	-3
x	x^2	$-3x$
-7	$-7x$	21

h. $3x^2 - 3$
$3(x^2 - 1)$
$3(x + 1)(x - 1)$

	x	-1
x	x^2	$-x$
1	x	-1

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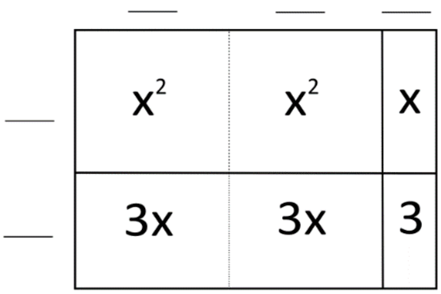
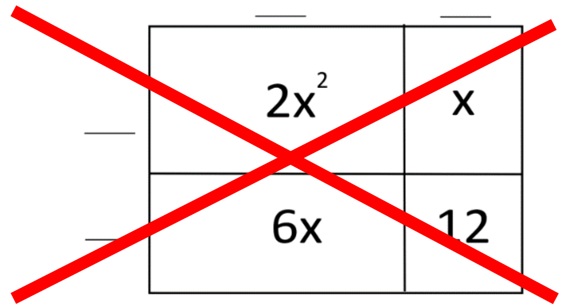
3. In this problem, students will analyze an area model for an expression where the leading coefficient is not a common factor.
    a. The sum of the areas is $2x^2 + 5x + 3$.
    b. There are two x^2 terms. The area is decomposed into more than four rectangles.
    c. $(2x + 3)(x + 1)$

4. Students should analyze each area model. While the totals may be equivalent, not all of the models correctly decompose the rectangle.

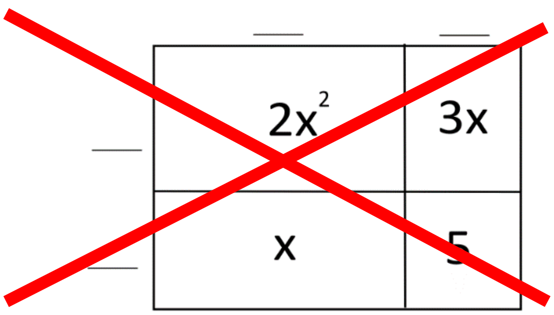
a. $2x^2 + 7x + 3 = (2x + 1)(x+3)$



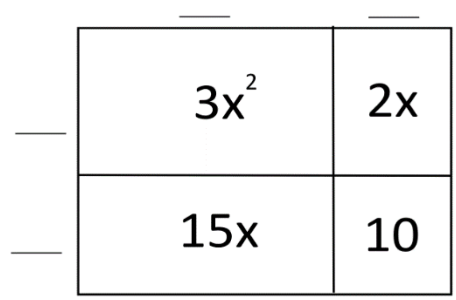
b. $2x^2 + 7x + 12$



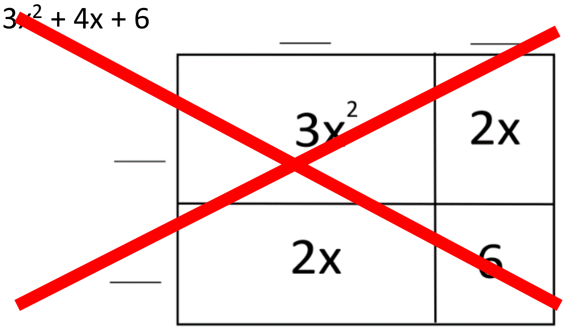
c. $2x^2 + 4x + 5$



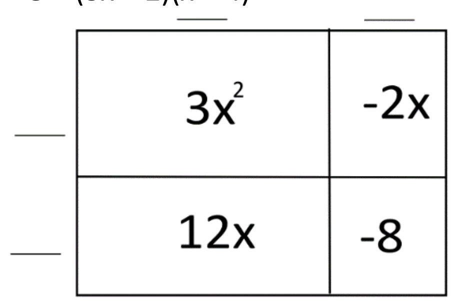
d. $3x^2 + 17x + 10 = (3x + 2)(x + 5)$



e. $3x^2 + 4x + 6$



f. $3x^2 + 10x – 8 = (3x – 2)(x + 4)$



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5.

a. $3x^2 + 4x + 1 = (3x + 1)(x + 1)$

	3x	1
x	$3x^2$	x
1	x	1

b. $2x^2 + 5x + 2 = (2x + 1)(x + 2)$

	2x	1
x	$2x^2$	x
2	2x	2

c. $7x^2 + 20x – 3 = (7x – 1)(x + 3)$

	7x	-1
x	$7x^2$	-x
3	21x	-3

d. $5x^2 - 14x – 3 = (5x + 1)(x – 3)$

	5x	1
x	$5x^2$	x
-3	-15x	-3

e. $4x^2 + 13x – 12 = (4x – 3)(x + 4)$

	4x	-3
x	$4x^2$	-3x
4	16x	-12

f. $2x^2 + 9x – 35 = (2x – 5)(x + 7)$

	2x	-5
x	$2x^2$	-5x
7	14x	-35

6. These practice problems are designed to mix the different strategies and have students become more fluent in factoring.

a. $4x^2 - 32x + 60$
$4(x^2 – 8x + 15)$
$4(x – 5)(x – 3)$

	x	-5
x	x^2	-5x
-3	-3x	15

b. $2x^2 + 13x + 15$
$(2x + 3)(x + 5)$

	2x	3
x	$2x^2$	3x
5	10x	15

c. $x^2 + 10x + 25 = (x + 5)^2$

	x	5
x	x^2	5x
5	5x	25

d. $4x^2 - 100 = 4(x^2 - 25) = 4(x + 5)(x - 5)$

	x	5
x	x^2	5x
-5	-5x	-25

e. $5x^2 - 30x + 45 = 5(x^2 - 6x + 9) = 5(x - 3)^2$

	x	-3
x	x^2	-3x
-3	-3x	9

Summarize the Mathematics

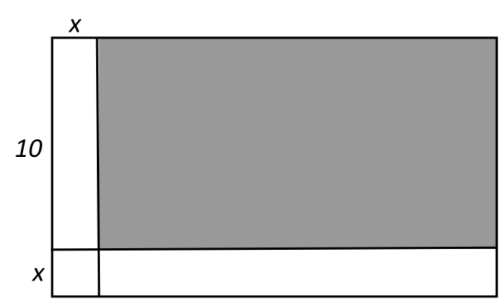
a. Student answers will vary. Monitor students as they work on the problem and facilitate a discussion on the approach students take. Will students begin with the two linear factors and then write the product? Will they create an area model and then write the expressions? Students could also exchange their expanded expressions and factor them.

b. Use the area model with one x^2 and determine factors of c that will add to get b.

c. Look for a common factor. If there is a common factor, use the method mentioned in part b. If not a common factor, reason with the area model using different number combinations.

Check Your Understanding

a. The area of the large rectangle has an area of $3x^2 + 38x + 80$. By factoring the expression into $(3x + 8)(x + 10)$, the length of the shaded rectangle is $2x + 8$ and the width is the given $x + 10$. Thus, the area of the shaded region is $10(2x + 8) = 20x + 80$.



b. Students may represent using the area model. Continue to encourage students to use the model if they are struggling to generalize.

i. $6x^2 + 6x - 12$
$6(x + 2)(x - 1)$

ii. $x^2 - 49$
$(x + 7)(x - 7)$

iii. $2x^2 - 5x - 3$
$(2x + 1)(x - 3)$

iv. $x^2 + 14x + 49$
$(x + 7)^2$

v. $5x^2 - 5x - 30$
$5(x - 3)(x + 2)$

vi. $3x^2 - 7x + 2$
$(3x - 1)(x - 2)$

vii. $x^2 - 20x + 100$
$(x - 10)^2$

viii. $4x^2 - 81$
$(2x + 9)(2x - 9)$