ICM Review ♦Unit 7 ♦ Probability Name Key Date _____

- A sample space of an experiment is the set of all possible outcomes.
- An event is a subset of the sample space of a given experiment.
- General Formula for Probability = $\frac{\text{Number of favorable outcomes}}{\text{Number of possible outcomes}}$
- If E and F are any two events of an experiment, then $P(E \cup F) = P(E) + P(F) - P(E \cap F)$.
- If E and F are mutually exclusive (meaning that only one of them can occur at one time), then $P(E \cup F) = P(E) + P(F)$.
- $P(B|A)$ = "probability of B given A" = $\frac{n(A \cap B)}{n(A)}$ or $\frac{P(A \cap B)}{P(A)}$
- $P(A \cap B)$ = "probability of A and B" = $P(A) \cdot P(B|A)$
- If two events are independent (meaning the outcome of one does not affect the other), then $P(A \cap B)$ = "probability of A and B" = $P(A) \cdot P(B)$

1. Let E and F be two **mutually exclusive** events and suppose P(E) = .4 and P(F) = .3. Compute:
   a) $P(E \cap F)$ = 0
   b) $P(E \cup F)$ = .4 + .3 = .7
   c) $P(E^c)$ = 1 − .4 = .6

2. Let E and F be two events and suppose P(E) = .35, P(F) = .55, and $P(E \cup F)$ = .7.
   a) Find $P(E \cap F)$.
   .7 = .35 + .55 − x
   x = .2
   b) Find P (E| F).
   $\frac{P(E \wedge F)}{P(F)} = \frac{.2}{.55} = .364$

3. A die is loaded, and it was determined that the probability distribution associated with the experiment of casting the die and observing which number falls uppermost is given by

Simple event	Probability
{1}	.20
{2}	.12
{3}	.16
{4}	.18
{5}	.15
{6}	.19

   a) What is the probability of the number being even? .12 + .18 + .19 = .49
   b) What is the probability of the number being either a 1 or a 6? .20 + .19 = .39
   c) What is the probability of the number being less than 4? .20 + .12 + .16 = .48

4. In a television game show, the winner is asked to select three prizes from 5 different prizes: A, B, C, D, and E. Describe a sample space of possible outcomes (order is not important). $n(S) = {}_5C_3 = 10$
   {ABC, ABD, ABE, ACD, ADE, BCD, BCE, BDE, ACE, CDE}

5. Eighty people were asked their favorite color, resulting in the data in the following table. Find the probability distribution associated with the data given below.

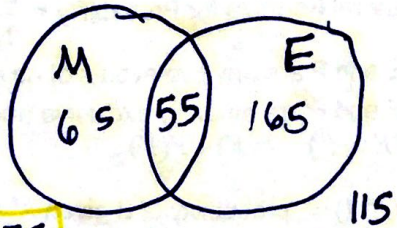
Color	Red	Orange	Yellow	Green	Blue	Purple
Frequency of Occurrence	14	12	10	14	17	13
	.175	.15	.125	.175	.2125	.1625

6. Of 400 college students, 120 are enrolled in math, 220 are enrolled in English, and 55 are enrolled in both. If a student is selected at random, find the probability that

a) the student is enrolled in mathematics. $\frac{120}{400} = .3$

b) the student is enrolled in mathematics or English. $\frac{285}{400} = .7125$

c) the student is enrolled in either math or English, but not both. $\frac{230}{400} = .575$



7. A survey of couples in a city found the following probabilities:
   - The probability that the husband is employed is 0.85.
   - The probability that the wife is employed is 0.60.
   - The probability that both are employed is 0.55.

   A couple is selected at random. Find the probability that

a) at least one of them is employed. $.85 + .60 - .55 = .9$

b) neither is employed. $.1$

8. A health and safety committee is to be selected from all the people who work at a local factory. The committee is to consist of four members selected randomly from a list of twelve names submitted by the shop leader. The list has the names of seven union members and five workers who are not union members.

a) What is the probability that exactly one person chosen for the committee is a union member?

$$\frac{C(7,1)\cdot C(5,3)}{C(12,4)} = \frac{7\cdot 10}{495} = \frac{14}{99} \text{ or } .141$$

b) What is the probability that at least one of the people chosen for the committee is a union member?

$$1 - P(0 \text{ Union}) = 1 - \frac{C(5,4)}{C(12,4)} = 1 - \frac{1}{99} = \frac{98}{99} \text{ or } .990$$

c) What is the probability that no more than 2 of the committee members are union members?

$$P(0,1,\text{or } 2) = \frac{C(5,4) + C(7,1)\cdot C(5,3) + C(7,2)\cdot C(5,2)}{C(12,4)} = \frac{285}{495} = \frac{19}{33} \text{ or } .576$$

9. The probability that Yeymi studies and makes an A on her math test is $\frac{17}{20}$. The probability that Yeymi studies is $\frac{15}{16}$. Find the probability that Yeymi makes an A on her math test, given that she has studied.

$$\frac{\frac{17}{20}}{\frac{15}{16}} = .907 \text{ or } \frac{68}{75}$$

10. Two cards are drawn in succession without replacement from a standard deck of 52 cards.

a) Find the probability that the first card is a face card and the second card is an ace.

$$\frac{12}{52}\cdot\frac{4}{51} = \frac{4}{221} \text{ or } .018$$

b) Find the probability that one is a face card and one is an ace.

$$\frac{C(12,1)\cdot C(4,1)}{C(52,2)} = \frac{8}{221} \text{ or } .036$$

c) Find the probability that both cards are face cards.

$$\frac{C(12,2)}{C(52,2)} = \frac{11}{221} \text{ or } .050$$

11. Here is a table showing trial results of a new acne cream:

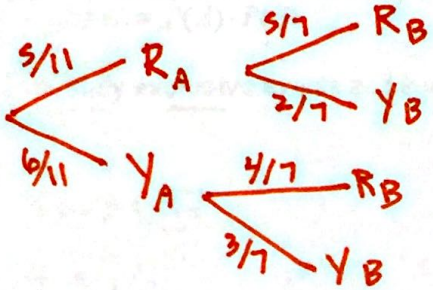
	Used Cream	Used Placebo
Skin Improved	800	600
No Improvement	400	200

a) Find the probability that someone's skin improved given that they used the acne cream. $\frac{800}{1200} = \frac{2}{3}$

b) If someone's skin improved, find the probability that they used the placebo. $\frac{600}{1400} = \frac{3}{7}$

12. Box A contains five red marbles and six yellow marbles. Box B contains four red marbles and two yellow marbles. A marble is drawn from Box A and then transferred to Box B. A marble is then drawn from Box B.

a) Draw a tree diagram that represents this situation with appropriate probabilities.



b) Find the probability that the transferred marble is yellow, given that the second marble is red?

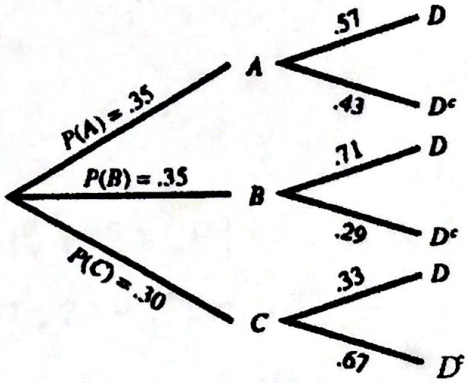
$$P(Y_A \mid R_B) = \frac{P(Y_A \cap R_B)}{P(R_B)} = \frac{\frac{6}{11}\cdot\frac{4}{7}}{\frac{5}{11}\cdot\frac{5}{7}+\frac{6}{11}\cdot\frac{4}{7}} = \frac{24}{49} \text{ or } .490$$

13. The diagram at the right represents an experiment consisting of two trials.

a) Find $P(A \cap D)$. $.35(.57) = .1995$

b) Find $P(D^c)$.

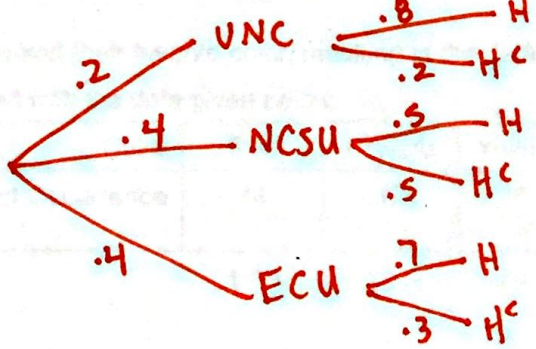
$.35(.43) + .35(.29) + .30(.67) = .453$



c) Find P(B|D).

$$\frac{P(B \cap D)}{P(D)} = \frac{.35(.71)}{1-.453} = .454$$

14. After college Ethan starts a business and hires only graduates of UNC, NC State, and ECU. He hires 20% from UNC, 40% from NC State, and 40% from ECU. Suppose 80% of UNC grads are honest, 50% of NC State grads are honest, and 70% of ECU grads are honest. After two years, he discovers someone has been embezzling funds. What is the probability that a Pirate is to blame?



$$P(ECU \mid H^c) = \frac{.4(.3)}{.2(.2)+.4(.5)+.4(.3)} = .333$$

Extra Practice

A box contains 6 blue marbles, 5 green, and 4 red marbles. Four marbles are selected from the box at random without replacement.

1. Find the probability that all four marbles are blue.

$$\frac{6c4}{15c4} = \frac{15}{1365} = \frac{1}{91} \text{ or } .011$$

2. Find the probability that exactly 2 marbles are blue. (and 2 are not blue)

$$\frac{6c2 \cdot 9c2}{15c4} = \frac{15*36}{1365} = \frac{36}{91} \text{ or } .396$$

3. Find the probability that at least 1 of the marbles is blue.

$$1 - P(\text{none blue}) = 1 - \frac{9c4}{15c4} = 1 - \frac{6}{65} = \frac{59}{65} \text{ or } .908$$

4. Find the probability that at most 2 are blue.

$$P(0, 1, \text{or } 2 \text{ are blue}) = \frac{9c4 + 6c1 \cdot 9c3 + 6c2 \cdot 9c2}{15c4} = \frac{1170}{1365}$$

$$= \frac{6}{7} \text{ or } .857$$

♦ Customers are asked to make a determination about their satisfaction following a visit to a convenience store. The following table reports how many new and returning customers are either satisfied or unsatisfied.

	Satisfied	Unsatisfied	
New	400	200	600
Returning	500	100	600
	900	300	1200

A customer is selected at random.

5. Find the probability that the customer is satisfied.

$$\frac{900}{1200} = \frac{3}{4} \text{ or } .75$$

6. Find the probability that the customer is a returning customer given that they are unsatisfied.

$$\frac{100}{300} = \frac{1}{3} \text{ or } .333$$

7. Find the probability that a new customer is satisfied.

$$\frac{400}{600} = \frac{2}{3} \text{ or } .667$$

♦ The diagram at the right represents an experiment consisting of two trials.

8. Find $P(A \cap D)$. $.35(.57) = .1995$

9. Find $P(D^c)$. $.35(.43) + .35(.29) + .30(.67)$

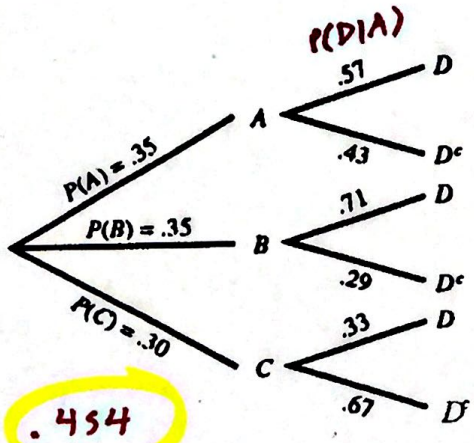
$.453$

10. Find $P(D|A)$.

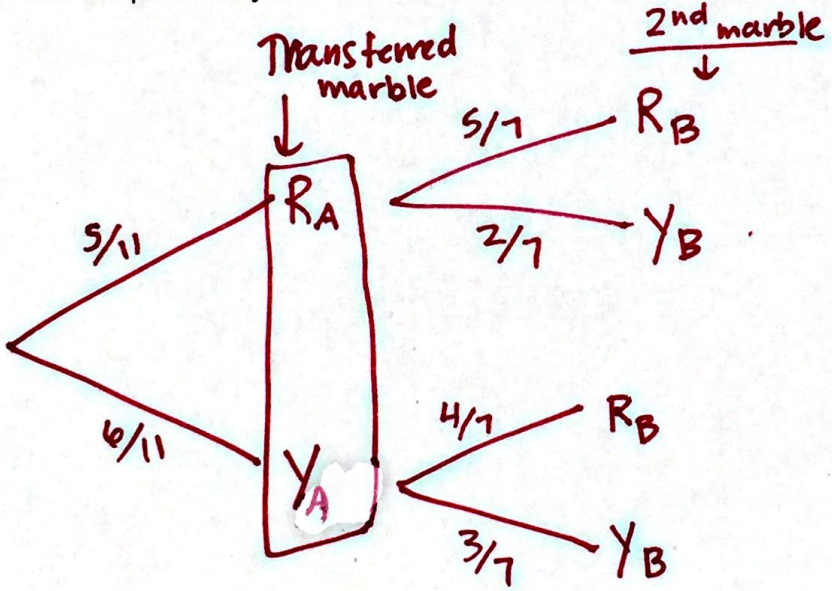
$.57$

11. Find $P(B|D)$. $= \frac{P(B \cap D)}{P(D)} = \frac{.35(.71)}{.547} = .454$

$P(D) = 1 - .453 = .547$



12. A Box A contains five red marbles and six yellow marbles. A Box B contains four red marbles and two yellow marbles. A marble is drawn from Box A and then transferred to Box B. A marble is then drawn from Box B. Draw a tree diagram that represents this situation with appropriate probabilities. Then find the probability that the transferred marble is yellow, given that the second marble is red?



$$P(Y_A \mid R_B) = \frac{\frac{6}{11} \cdot \frac{4}{7}}{\frac{5}{11} \cdot \frac{5}{7} + \frac{6}{11} \cdot \frac{4}{7}} = \frac{24}{49}$$

$$= .490$$