

Unit ³/₁ * Probability Distributions and Statistics

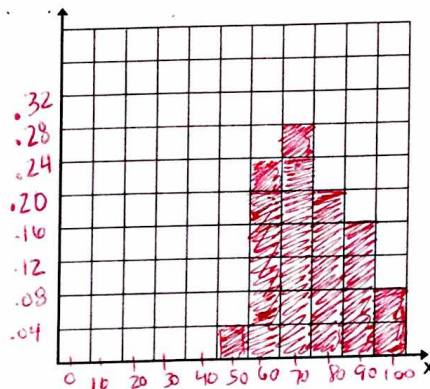
1. A class of 25 students took a 10-point math quiz. The following frequency distribution describes the scores received on the quiz.

Score	50	60	70	80	90	100
Frequency	1	6	7	5	4	2

- a) Find the probability distribution of X, where X denotes the score received on the quiz.

X	50	60	70	80	90	100
P(X=X)	.04	.24	.28	.2	.16	.08

- b) Compute the mean to the nearest tenth.
 c) Compute the standard deviation to the nearest tenth.
 d) Draw a histogram to represent the probability distribution.



2. A game is considered fair when the expected value is 0. What should you pay to play a game in which you receive \$5 for drawing an ace from a deck of cards, \$1 for a face-card, and 50¢ for any other card if the game is to be a fair game?

$$5\left(\frac{4}{52}\right) + 1\left(\frac{12}{52}\right) + .50\left(\frac{36}{52}\right) = \boxed{\$.96}$$

3. A local marathon runner estimates that the probability he will win his next race is 0.4.

- a) What are the odds that he will win his next race? 4:6 or 2:3
 b) What are the odds that he will not win his next race? 6:4 or 3:2

4. A casino advertises that the odds of winning one of its games are 4 to 7. What is the probability of winning this game?

$$\frac{4}{11} \text{ or } .36$$

5. A box contains 6 half dollars, 3 quarters, 10 dimes, and 5 nickels. A coin is drawn at random from the box.

a) Calculate the mean of the value of the draw to the nearest cent. (At least write down your probability distribution. Otherwise, partial credit for mistakes cannot be given.)

X	.5	.25	.10	.05
P(X=X)	6	3	10	5

$\boxed{\$.21}$

- b) Calculate the standard deviation of the value of the draw to the nearest cent.

$\boxed{\$.18}$

6. A baseball player has a 0.210 batting average.

- a) How many hits would you expect this player to get in 50 times at bat?

$$50(.210) = \boxed{10.5 \text{ hits}}$$

- b) Calculate the standard deviation.

$$\sqrt{npq} = \sqrt{50(.210)(.79)} = \boxed{2.88}$$

$q = 1 - p$

7. The probability that a marksman shooting at a target will hit a bull's-eye is 0.15. What is the probability that he will hit at most 1 bull's-eye in 8 shots? (Round to the nearest thousandth.)

$$\text{binomial pdf}(8, .15, 0) + \text{binomial pdf}(8, .15, 1) = .2725 + .3847 = \boxed{.657}$$

8. The probability that a bowler will be left with a 7-10 split after bowling his first ball in each frame is 0.1.

- a) What is the probability that he will get exactly two 7-10 splits in 5 frames? (Round to the nearest thousandth.)

$$\text{binomial pdf}(5, .1, 2) = \boxed{.073}$$

- b) What is the probability that he will get at least one 7-10 split in 10 frames? (Round to the nearest thousandth.)

$$1 - P(X=0) = 1 - \text{binomial pdf}(10, .1, 0) = 1 - .349 = \boxed{.651}$$

9. If the Stanford-Binet IQ test is a normal curve with mean of 100 and standard deviation of 20, what percent of the population has an IQ below 85?

$$P(X < 85) = \text{normalcdf}(-10, 99, 85, 100, 20) = \boxed{.227}$$

10. The birth weights of babies born in a certain hospital has been found to be normally distributed with a mean of 7.4 pounds and a standard deviation of 1.2 pounds.


- a) Find the probability that the birth weight of a baby born in this hospital is more than 8.5.

$$P(X > 8.5) = \boxed{.180}$$

- b) Find the probability that the birth weight of a baby born in this hospital is between 6 and 9 pounds.

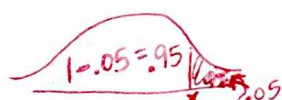
$$P(6 < X < 9) = \boxed{.787}$$

- c) Find the weight of babies born at this hospital in the 90th percentile in birth weight.



$$\text{invNorm}(.90, 7.4, 1.2) = \boxed{8.94 \text{ lbs}}$$

11. In a biology class of 150 students at Hampton University, Unit 2 test scores had a mean of 75 and a standard deviation of 9. If the professor were to adjust the scores to a normal curve so that only 5% of her students get A's on that test, what is the minimum score to receive an A?



$$\text{invNorm}(.95, 75, 9) = 89.8 \text{ or } 90$$

12. The expected life span of a battery is normally distributed with a mean of 50 hours and a standard deviation of 6 hours. The manufacturer advertises that they will replace all batteries that last less than 38 hours. If 50,000 batteries were produced, how many would they expect to replace?

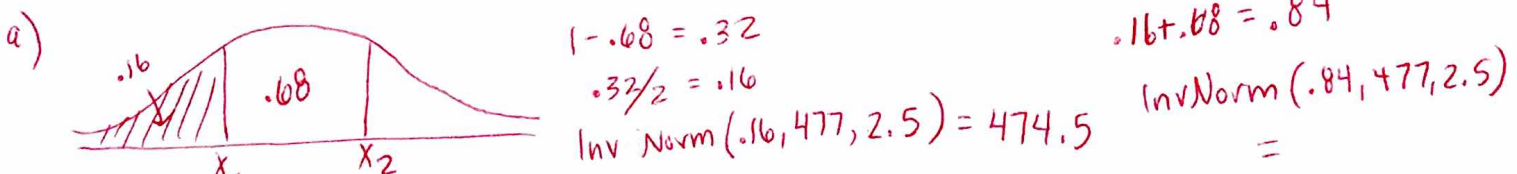
$$P(X < 38) = .023 \quad .023(50,000) = \boxed{1138 \text{ batteries}}$$

13. The machine that fills bottles of fruit punch is set so that the mean volume is 477 ml. A bottle is considered full if it contains at least 473 ml. The volumes in the bottles are normally distributed with a standard deviation of 2.5 ml.

- a) What percent of the bottles are under filled? $P(X < 473) = .055$ so $\boxed{5.5\%}$

- b) 68% of the bottles contain between 474.5 ml and 479.5 ml.

- c) 95% of the bottles contain between 472 ml and 482 ml.



14. A set of mathematics exam scores has a mean of 70 and a standard deviation of 8. A set of English exam scores has a mean of 74 and a standard deviation of 16. For which exam would a score of 78 have a higher standing?

Math $\frac{78-70}{8} = 1$

English $\frac{78-74}{16} = .25$

15. A distribution of scores has a standard deviation of 10. Find the z-scores corresponding to the following values:

- a) A score that is 20 points below the mean

-2

- b) A score that is 15 points above the mean

1.5

16. A normal distribution has a mean of 120 and a standard deviation of 20. For this distribution, what score corresponds to the 90th percentile?

$\text{invNorm}(.9, 120, 20) = 145.6$

17. On a multiple-choice test, a student is given five possible answers for each question. The student receives 1 point for a correct answer and loses $\frac{1}{4}$ point for an incorrect answer. If the student has no idea of the correct answer for a particular question and merely guesses, what is the student's expected gain or loss on the question?

X	1	$-\frac{1}{4}$	= 0 0
P(X=x)	$\frac{1}{5}$	$\frac{4}{5}$	

18. Suppose the warranty period on your family's new television is about to expire and you are debating about whether to buy a one-year maintenance contract for \$35. If you buy the contract, all repairs for one year are free. Consumer information shows that 12% of the televisions like yours require an annual repair that costs \$140 on the average. Would you advise buying the maintenance contract? Explain your reasoning.

$140(.12) = 16.8$ $-35 + 16.8 = -18.2$

No. The expected cost of repair is less than the cost of the maintenance contract.

19. The Palm Coast investment club is considering purchasing a certain stock. After considerable research, the club members determine that there is a 60% chance of making \$8000, a 10% chance of breaking even and a 30% chance of losing \$6200. Find the expected value of this purchase.

$.6(8000) + .1(0) + .3(-6200) = 2940$