Name ______________________________ Date ____________

7.2 Exercises

The Definition of Derivative

Use the definition of derivative to find $f'(x)$.

1. $f(x) = 2x^2 + x - 1$
2. $f(x) = 1 - x^2$
3. $f(x) = x^3 - 12x$
4. $f(x) = x^3 + x^2$
5. $f(x) = \dfrac{1}{x-1}$
6. $f(x) = \dfrac{3}{x+2}$
7. $f(x) = \dfrac{1}{x^2}$
8. $f(x) = \sqrt{x-4}$

Write the equation of the tangent line to the graph of f at the indicated point.

9. $f(x) = 2x^2 + x - 1$ at (1, 2)
10. $f(x) = \dfrac{1}{x-1}$ at (0, -1)

1. $$f'(x) = \lim_{h \to 0} \frac{2(x+h)^2 + (x+h) - 1 - (2x^2 + x - 1)}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{2x^2} + 4xh + 2h^2 + \cancel{x} + h \cancel{- 1} \cancel{- 2x^2} \cancel{- x} \cancel{+ 1}}{h} = \lim_{h \to 0} \frac{4x\cancel{h} + 2h^{\cancel{2}} + \cancel{h}}{\cancel{h}}$$

$$= \lim_{h \to 0} 4x + 2h + 1$$

$$= 4x + 2(0) + 1 \qquad \boxed{f'(x) = 4x + 1}$$

2. $$f'(x) = \lim_{h \to 0} \frac{1 - (x+h)^2 - (1 - x^2)}{h}$$

$$= \lim_{h \to 0} \frac{1 - (x^2 + 2xh + h^2) - 1 + x^2}{h} = \lim_{h \to 0} \frac{\cancel{1} \cancel{- x^2} - 2xh - h^2 \cancel{- 1} \cancel{+ x^2}}{h}$$

$$= \lim_{h \to 0} \frac{-2x\cancel{h} - h^{\cancel{2}}}{\cancel{h}} \qquad \boxed{f'(x) = -2x}$$

$$= \lim_{h \to 0} -2x - (0)$$

3. $f'(x)=\lim\limits_{h\to 0} \dfrac{(x+h)^3-12(x+h)-(x^3-12x)}{h}$

$=\lim\limits_{h\to 0} \dfrac{(x^2+2xh+h^2)(x+h)-12x-12h-x^3+12x}{h}$

$=\lim\limits_{h\to 0} \dfrac{\cancel{x^3}+hx^2+2x^2h+2xh^2+h^2x+h^3-\cancel{12x}-12h-\cancel{x^3}+\cancel{12x}}{h}$

$=\lim\limits_{h\to 0} \dfrac{3\cancel{h}x^2+3xh^2+h^3-12\cancel{h}}{\cancel{h}} = \lim\limits_{h\to 0} 3x^2+3xh+h^2-12$

$=\lim\limits_{h\to 0} 3x^2+3x(0)+(0)^2-12$ $\quad \boxed{f'(x)=3x^2-12}$

4. $f'(x)=\lim\limits_{h\to 0} \dfrac{(x+h)^3+(x+h)^2-(x^3+x^2)}{h}$

$=\lim\limits_{h\to 0} \dfrac{\cancel{x^3}+hx^2+2x^2h+2xh^2+h^2x+h^3+\cancel{x^2}+2xh+h^2-\cancel{x^3}-\cancel{x^2}}{h}$

$=\lim\limits_{h\to 0} \dfrac{3x^2\cancel{h}+3xh^2+h^3+2x\cancel{h}+h^2}{\cancel{h}}$ $\quad \boxed{f'(x)=3x^2+2x}$

$=\lim\limits_{h\to 0} 3x^2+3xh+h^2+2x+h$

$=3x^2+3x(0)+(0)^2+2x+(0)$

5. $f'(x)=\lim\limits_{h\to 0} \dfrac{\dfrac{1}{(x+h)-1}-\dfrac{1}{x-1}}{h} = \lim\limits_{h\to 0} \dfrac{(x-1)\dfrac{1}{(x-1)(x+h-1)}-\dfrac{1\,(x+h-1)}{(x-1)(x+h-1)}}{h}$

$=\lim\limits_{h\to 0} \dfrac{\dfrac{x-1}{(x-1)(x+h-1)}-\dfrac{(x+h-1)}{(x-1)(x+h-1)}}{h} = \lim\limits_{h\to 0} \dfrac{\dfrac{\cancel{x}-\cancel{1}-\cancel{x}-h+\cancel{1}}{(x-1)(x+h-1)}}{h}$

$=\lim\limits_{h\to 0} \dfrac{-\cancel{h}}{(x-1)(x+h-1)}\cdot\dfrac{1}{\cancel{h}} = \dfrac{-1}{(x-1)(x+(0)-1)}$

$\boxed{f'(x)=\dfrac{-1}{(x-1)^2}}$

$$6.\quad f'(x) = \lim_{h\to 0} \frac{\frac{3}{(x+h+2)} - \frac{3}{x+2}}{h} = \lim_{h\to 0} \frac{\frac{(x+2)3}{(x+2)(x+h+2)} - \frac{3(x+h+2)}{(x+2)(x+h+2)}}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{3x}+\cancel{6}-\cancel{3x}-3h-\cancel{6}}{(x+2)(x+h+2)} \cdot \frac{1}{\cancel{h}}\ h$$

$$= \lim_{h\to 0} \frac{-3\cancel{h}}{(x+2)(x+h+2)} = \frac{-3}{(x+2)(x+0+2)}$$

$$\boxed{f'(x) = \frac{-3}{(x+2)^2}}$$

$$7.\quad f'(x) = \lim_{h\to 0} \frac{\frac{1}{(x+h)^2} - \frac{1}{x^2}}{h} = \lim_{h\to 0} \frac{\frac{(x^2)1}{(x^2)(x+h)^2} - \frac{1}{x^2}\cdot\frac{(x+h)^2}{(x+h)^2}}{h}$$

$$= \lim_{h\to 0} \frac{\frac{x^2}{x^2(x+h)^2} - \frac{(x+h)^2}{x^2(x+h)^2}}{h} = \lim_{h\to 0} \frac{\cancel{x^2}-\cancel{x^2}-2x\cancel{h}-h^2}{x^2(x+h)^2}\cdot\frac{1}{\cancel{h}}$$

$$= \lim_{h\to 0} \frac{-2x-h}{x^2(x+h)^2} = \frac{-2x-(0)}{x^2(x+0)^2} = \frac{-2x}{x^2\cdot x^2} = \frac{-2x^1}{x^4} = \frac{-2}{x^3}$$

$$\boxed{f'(x) = \frac{-2}{x^3}}$$

$$8.\quad f'(x) = \lim_{h\to 0} \frac{\sqrt{(x+h)-4} - \sqrt{x-4}}{h} \cdot \frac{(\sqrt{(x+h-4}+\sqrt{x-4})}{(\sqrt{(x+h-4}+\sqrt{x-4})}$$

$$= \lim_{h\to 0} \frac{(\cancel{x}+\cancel{h})-\cancel{4}-\cancel{x}+\cancel{4}}{h(\sqrt{x+h-4}+\sqrt{x-4}} \cdot \frac{1}{\cancel{h}} = \frac{1}{\sqrt{x+(0)-4}+\sqrt{x-4}}$$

$$= \lim_{h\to 0} \frac{1}{\sqrt{x-4}+\sqrt{x-4}} = \boxed{f'(x) = \frac{1}{2\sqrt{x-4}}}$$