Union and Intersection of Sets

A.A.31 Find the intersection of sets (no more than three sets) and/or union of sets (no more than three sets).

Check Skills You’ll Need

Write each set in set-builder notation.
1. \( A = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\} \)
2. \( B = \{1, 3, 5, 7\} \)

Write each set in roster form.
3. \( C = \{n \mid n \text{ is an even number between } -15 \text{ and } -5\} \)
4. \( D = \{k \mid k \text{ is a composite number between } 7 \text{ and } 17\} \)

New Vocabulary
- union
- intersection
- disjoint sets

Operations on Sets

In Lesson NY-3, you examined different ways to express sets. In this lesson, you will perform operations on these sets.

The union of two or more sets is the set that contains all elements of the sets. The symbol for union is \( \cup \). To find the union of two sets, list the elements that are in either set, or in both sets. In the Venn diagram below, \( A \cup B \) is shaded.

EXAMPLE

Union of Sets

A bouquet of flowers contains roses, carnations, and baby’s breath. A second bouquet has roses, lilies, and daisies. Both bouquets are put in the same vase. Use union of sets to find the set of flowers in the vase.

first bouquet: \( B = \{\text{roses, carnations, baby’s breath}\} \)
second bouquet: \( S = \{\text{roses, lilies, daisies}\} \)

List the flowers that are in either bouquet, or in both bouquets.

\( B \cup W = \{\text{roses, carnations, baby’s breath, lilies, daisies}\} \)

Quick Check

1. \( P = \{5, 10, 15, 20\} \) and \( Q = \{8, 10, 18, 20\} \). Find \( P \cup Q \).
Vocabulary Tip

A ∩ B is read “A intersection B,” or “the intersection of sets A and B.”

The intersection of sets is the set of elements that are common to two or more sets. The symbol for intersection is ∩. When you find the intersection of two sets, list only the elements that are in both sets. The shaded area below shows A ∩ B.

Disjoint sets have no elements in common. The intersection of disjoint sets is the empty set. The diagram below shows two disjoint sets.

2 Example Intersection of Sets

Set C = \{x | x \text{ is a natural number less than 20}\}, set D = \{y | y \text{ is an odd integer}\}, and set E = \{z | z \text{ is a multiple of 4}\}.

a. Find C ∩ D.

List the elements that are both odd integers and natural numbers less than 20.

C ∩ D = \{1, 3, 5, 7, 9, 11, 13, 15, 17, 19\}

b. Find C ∩ E.

List the elements that are both multiples of 4 and natural numbers less than 20.

C ∩ E = \{4, 8, 12, 16\}

c. Find D ∩ E.

D ∩ E = \emptyset, or the empty set. There are no multiples of 4 that are also odd, so these are disjoint sets. They have no elements in common.

Quick Check 2 X = \{2, 4, 6, 8, 10\}, Y = \{0, 2, 5, 7, 8\}, and Z = \{n | n \text{ is an odd integer}\}.

a. Find X ∩ Y. b. Find X ∩ Z. c. Find Z ∩ Y.

Solving Problems With Venn Diagrams

You can use Venn diagrams to show the relationships between sets. You can write the elements inside the appropriate section of the diagram. Elements in the intersection of sets appear in the overlapping sections of the Venn diagram.
Lesson NY-4  Union and Intersection of Sets

**Venn Diagrams: Three Sets**

**Presidents** The set of presidents on Mount Rushmore can be represented by the set $R = \{\text{Washington, Jefferson, Lincoln, T. Roosevelt}\}$. The set of presidents born in February can be represented by the set $F = \{\text{Washington, Harrison, Lincoln, Reagan}\}$. Presidents who had beards can be represented by the set $B = \{\text{Lincoln, Grant, Garfield, Harrison}\}$. Draw a Venn diagram to represent the union and intersection of these sets.

Draw and label three intersecting circles to represent the sets.

Note that each element only appears in the Venn diagram in one location.

**Quick Check**

Let $X = \{a, e, i, o, u\}$, $Y = \{a, b, c, d, e, f\}$, and $Z = \{a, c, e\}$. Draw a Venn diagram to represent the intersection and union of the sets.

You can also use Venn diagrams to show the number of elements in the union or intersection of a set.

**Real-World Connection**

**Pets** Of 50 cat and dog owners surveyed, 25 have a cat. Ten owners have a dog and a cat. How many owners have a dog?

**Step 1** Draw a Venn diagram. Let $C = \text{owners with cats}$ and $D = \text{owners with dogs}$.

**Step 2** The intersection of $C$ and $D$ represents the owners with both cats and dogs. $C \cap D = 10$.

**Step 3** Find the number of owners with only a cat. $25 - 10 = 15$. Enter 15 into the Venn diagram.

**Step 4** The total number of owners is 50. Subtract to find the number of owners with only a dog. $50 - 15 - 10 = 25$.

- The total number of dog owners is $25 + 10 = 35$.

**Quick Check**

Of 240 college freshmen, 152 are taking history and 81 are taking science and history. How many freshmen are taking history but not science?
For Exercises 1–12, find each union or intersection. Let $A = \{2, 5\}$, $B = \{5, 7, 9\}$, $C = \{x \mid x$ is an odd number less than 9\}$, and $D = \{x \mid x$ is an even number less than 9\}$.

1. $A \cup B$
2. $A \cup C$
3. $A \cup D$
4. $B \cup C$
5. $B \cup D$
6. $C \cup D$
7. $A \cap B$
8. $A \cap C$
9. $A \cap D$
10. $B \cap C$
11. $B \cap D$
12. $C \cap D$

For Exercises 13–16, draw a Venn diagram to represent the union and intersection of the given sets.

13. **Pets** Alex has cats, rabbits, and fish as pets. Becky has cats and dogs. Cory has cats, birds, fish, and turtles. Let $A = \{$cats, rabbits, fish$, B = \{$cats, dogs$, and $C = \{$cats, birds, fish, turtles$\}$.

14. Let $X = \{x \mid x$ is a letter in the word ALGEBRA$, $Y = \{y \mid y$ is a letter in the word GEOMETRY$, and $Z = \{z \mid z$ is a letter in the word CALCULUS\}$.

15. Let $P = \{x \mid x$ is a prime number less than 10\}$, $C = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, and $O = \{a \mid a$ is an odd number less than 10\}$.

16. Let $L = \{a, b, c, 1, 2, 3, horse, cow, pig\}$, $M = \{-1, 0, 1, b, y, pig, duck, \Delta\}$, and $N = \{c, 3, duck, \Delta\}$.

17. Of 75 people with cell phones, 42 take pictures with their cell phone and 36 use their cell phone to take pictures and send text messages. How many use their phones to send text messages?

18. Of 100 people in a band, 70 members said they play a sport and 50 members play a sport and take music lessons. How many take music lessons?

19. An ice cream shop owner surveys 200 people who eat chocolate and vanilla ice cream. If 154 people like both flavors, and 196 people like vanilla, how many people like chocolate?

20. **Reasoning** Find two sets $A$ and $B$ such that $A \cup B = \{1, 2, 3, 4, 5\}$ and $A \cap B = \{2\}$.

21. **Writing** Let $M = \{x \mid x$ is a multiple of 3\}$ and $N = \{x \mid x$ is a multiple of 4\}$. Describe the intersection of $M$ and $N$.

22. **Critical Thinking** Set $X$ has 10 elements, set $Y$ has 15 elements, and $X \cap Y$ has 5 elements. How many elements are in $X \cup Y$? Explain.

For Exercises 23–25, identify each statement as true or false. Use a Venn diagram or give a counterexample to justify your answer.

23. The intersection of two sets is always a subset of their union.

24. Two sets that contain no elements in common are disjoint sets.

25. The intersection of the set of even numbers and the set of prime numbers is the empty set.
26. Of students surveyed, 80 had visited the planetarium, 75 had visited Ellis Island, and 65 had been to Niagara Falls. 30 students went to both the planetarium and Ellis Island, 25 went to the planetarium and Niagara Falls, and 10 went to Ellis Island and Niagara Falls. Only 5 went to all three.
   a. How many students went only to the planetarium?
   b. How many students went only to Niagara Falls?
   c. How many students did not go to Ellis Island but went to the planetarium or Niagara Falls?
   d. How many total students were surveyed?

27. **Open-Ended** Find two sets $A$ and $B$ such that $A \cup B = \{1, 2, 3, 4, 5\}$ and $A \cap B = \emptyset$.

28. Let $P = \{x \mid x$ is a prime number$\}$ and $Q = \{x \mid x$ is a multiple of 6$\}$. Describe the intersection of $P$ and $Q$.

For Exercises 29–32, write each set in roster notation. Let $X = \{a, c, e, g, i\}$, $Y = \{b, c, e, f\}$, and $Z = \{b, d, f\}$.

29. $Z \cap (X \cup Y)$
30. $Z \cup (X \cap Y)$
31. $Y \cup (Z \cap X)$
32. $(Y \cup Z) \cap X$

33. **Critical Thinking** Is the equality $(A \cup B) \cap C = A \cup (B \cap C)$ always, sometimes, or never true? Justify your answer.

34. Consider the set $D$ of all numbers in the interval $(-\infty, 5)$. What is the complement of $D$?
   A. $(-\infty, -5)$
   B. $[5, +\infty)$
   C. $(-\infty, 5)$
   D. $(5, +\infty)$

35. Set $X = \{x \mid x$ is a factor of 12$\}$ and set $Y = \{y \mid y$ is a factor of 16$\}$. Which set represents $X \cap Y$?
   F. $\emptyset$
   G. $\{0, 1, 2, 4\}$
   H. $\{1, 2, 4\}$
   J. $\{1, 2, 3, 4, 6, 8, 12, 16\}$

36. Let the universal set $U$ represent the integers from 1 to 10. Use the subsets of $M = \{x \mid x$ is a factor of 8$\}$, and $N = \{1, 4, 6, 7, 10\}$ to find the complement of $M \cup N$.

37. number of movies watched
38. favorite pizza toppings
39. colors of passing cars
40. SAT math scores

**Lesson NY-1**

**Lesson NY-2**

Classify each as qualitative or quantitative data.

Classify each as qualitative or quantitative data.

**Lesson NY-4**

Find the median, the first quartile, and the third quartile of each data set.

37. number of movies watched
38. favorite pizza toppings
39. colors of passing cars
40. SAT math scores

41. 3 5 8 2 11 15 13
42. 17 20 11 34 91 12 29
43. 105 121 151 89 93 141