ICM – Unit 6 Review   Name KEY   Date

You may use this review on your test. You may NOT photo copy anyone else's or print and glue anything on this review. All answer must be in YOUR handwriting. You will turn in this packet with your test.

Verify each trig identity.

1. $\tan x \sin x + \cos x = \sec x$

$\frac{\sin x}{\cos x} \cdot \sin x + \cos x = \frac{\sin^2 x}{\cos x} + \left(\frac{\cos x}{1}\right)\frac{\cos x}{\cos x}$

$\frac{\sin^2 x + \cos^2 x}{\cos x} = \frac{1}{\cos x} = \boxed{\sec x}$ ✓

2. $\frac{1}{\tan x} + \tan x = \frac{1}{\sin x \cos x}$

$\frac{1}{\frac{\sin x}{\cos x}} + \frac{\sin x}{\cos x} = \frac{\cos x}{\cos x}\left(\frac{\cos x}{\sin x}\right) + \left(\frac{\sin x}{\cos x}\right)\frac{\sin x}{\sin x} = \frac{\cos^2 x}{\cos x \sin x} + \frac{\sin^2 x}{\cos x \cdot \sin x}$

$= \boxed{\frac{1}{\cos x \cdot \sin x}}$ ✓

3. $\sin x - \sin x \cos^2 x = \sin^3 x$

$x - xy$

$\sin x(1-\cos^2 x) = \sin^3 x$

$\sin x(\sin^2 x) = \sin 3x$

$= \boxed{\sin^3 x}$ ✓

4. $\frac{\cos\alpha}{\cos\alpha}\left(\frac{\cos \alpha}{1+\sin \alpha}\right) + \left(\frac{1+\sin \alpha}{\cos \alpha}\right)\frac{(1+\sin\alpha)}{(1+\sin\alpha)} = 2\sec \alpha$

$\frac{\cos^2\alpha}{\cos\alpha(1+\sin\alpha)} + \frac{1+2\sin\alpha+\sin^2\alpha}{\cos\alpha(1+\sin\alpha)} = \frac{2+2\sin\alpha}{\cos\alpha(1+\sin\alpha)} = \frac{2(1+\sin\alpha)}{\cos\alpha(1+\sin\alpha)}$

$\frac{2}{\cos\alpha} = 2\cdot\frac{1}{\cos\alpha} = \boxed{2\sec\alpha}$ ✓

5. $\sin^4 x - \cos^4 x = 1 - 2\cos^2 x$

$(x^2-y^2) = (x+y)(x-y)$

$(\sin^2 x - \cos^2 x)(\sin^2 x + \cos^2 x)$

$(1-\cos^2 x - \cos^2 x)(\cancel{\sin^2 x} + (1 - \cancel{\sin^2 x}))$

$(1-2\cos^2 x)1 = \boxed{1-2\cos^2 x}$ ✓

$x^4 - y^4 = 0$

$(x-y)(x+y) = 0$

6. $(\sin x - \cos x)^2 + (\sin x + \cos x)^2 = 2$

$= (\sin x - \cos x)(\sin x - \cos x) + (\sin x + \cos x)(\sin x + \cos x)$

$= \underbrace{\sin^2 x - \cancel{2\cos x \sin x} + \cos^2 x}_{} + \underbrace{\sin^2 x + \cancel{2\cos x \sin x} + \cos^2 x}_{}$

$= 1 + 1 = \boxed{2}$ ✓

Use the sum and difference formulas to find the <u>exact</u> value of each. Show work!

7. tan 75° (u) (v)

$\tan(45+30) =$

$$= \frac{\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2}+\frac{1}{2}\cdot\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2}-\frac{\sqrt{2}}{2}\cdot\frac{1}{2}} = \frac{\frac{\sqrt{6}}{4}+\frac{\sqrt{2}}{4}}{\frac{\sqrt{6}}{4}-\frac{\sqrt{2}}{4}} = \frac{\sqrt{6}+\sqrt{2}}{\cancel{4}}\cdot\frac{\cancel{4}}{\sqrt{6}-\sqrt{2}} = \frac{\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}\cdot\frac{(\sqrt{6}+\sqrt{2})}{(\sqrt{6}+\sqrt{2})}$$

$$= \frac{\sqrt{36}+\sqrt{12}+\sqrt{12}+\sqrt{4}}{\sqrt{36}-\sqrt{4}} = \frac{8+2\sqrt{3}+2\sqrt{3}}{4} = \frac{8+4\sqrt{3}}{4} = \boxed{2+\sqrt{3}}$$

8. cos 195° (u) (v)

$$\cos(150+45) = -\frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2} - \frac{1}{2}\cdot\frac{\sqrt{2}}{2}$$

$$= \frac{-\sqrt{6}}{4} - \frac{\sqrt{2}}{4} = \boxed{\frac{-\sqrt{6}-\sqrt{2}}{4}}$$

9. $\sin\frac{\pi}{12}$

$$\sin(15°) = \sin(45-30) = \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} - \frac{1}{2}\cdot\frac{\sqrt{2}}{2}$$

$$= \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} = \boxed{\frac{\sqrt{6}-\sqrt{2}}{4}}$$

Prove each trig identity.

10. $\cos\left(x+\frac{\pi}{6}\right) - \sin\left(x+\frac{2\pi}{3}\right) = 0$ (u)(v) (u)(v)

$$\left(\cos x\cdot\frac{\sqrt{3}}{2} - \sin x\cdot\frac{1}{2}\right) - \left(\sin x\cdot -\frac{1}{2} + \frac{\sqrt{3}}{2}\cdot\cos x\right)$$

$$\cancel{\frac{\sqrt{3}}{2}\cos x} - \cancel{\frac{1}{2}\sin x} + \cancel{\frac{1}{2}\sin x} - \cancel{\frac{\sqrt{3}}{2}\cos x}$$

$\boxed{=0}$ ✓

11. $\frac{\sin(x-y)}{\cos x\cos y} = \tan x - \tan y$

$$\frac{\sin x\cdot\cos y - \sin y\cdot\cos x}{\cos x\cdot\cos y} = \frac{\sin x\,\cancel{\cos y}}{\cos x\cdot\cancel{\cos y}} - \frac{\sin y\cdot\cancel{\cos x}}{\cancel{\cos x}\cdot\cos y}$$

$$= \frac{\sin x}{\cos x} - \frac{\sin y}{\cos y}$$

$$= \boxed{\tan x - \tan y}$$

Prove each trig identity.

12. $(\sin x + \cos x)^2 - 1 = \sin 2x$

$(\sin x + \cos x)(\sin x + \cos x) - 1$

$\sin^2 x + 2\sin x \cdot \cos x + \cos^2 x - 1$

$\sin^2 x - 1 + 2\sin x \cos x + \cos^2 x$

$1 - 1 + 2\sin x \cdot \cos x$

$= 2\sin x \cdot \cos x = \boxed{\sin 2x}$ ✓

Solve for the variable on the interval $0 \le x < 2\pi$.

13. $4\cos^2 x - 3 = 0$

$+3 \quad +3$

$\frac{4\cos^2 x}{4} = \frac{3}{4}$

$\frac{\sqrt{3}}{\sqrt{4}}$

$\cos^2 x = \pm\frac{\sqrt{3}}{2}$

$\cos x = \pm\frac{\sqrt{3}}{2}$

$\boxed{x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}}$

14. $\cos^3 x = \cos x$

$x^3 - x = 0$

$x(x^2 - 1)$

$\cos^3 x - \cos x = 0$

$\cos x(\cos^2 x - 1) = 0$

$\cos x = 0 \quad | \quad \cos^2 x = 1$

$\cos x = \pm 1$

$\boxed{x = \pi/2, \frac{3\pi}{2}}$ $\quad$ $\boxed{x = 0\pi, \pi, 2\pi}$

15. $2\sin^2 x - \sin x - 1 = 0$

$(2x+1)(x-1)$

$2\sin x + 1 = 0 \quad | \quad \sin x - 1 = 0$

$-1 \; -1 \qquad +1 \; +1$

$\frac{2\sin x}{2} = \frac{-1}{2} \qquad \sin x = 1$

$\sin x = -1/2$

$\boxed{x = 7\pi/6, \frac{11\pi}{6}, \pi/2}$

16. $\sec^2 x - 2\tan x = 4$

$(1 + \tan^2 x) - 2\tan x = 4$

$x^2 + 2x + 3 = 0 \quad 1 + \tan^2 x - 2\tan x = 4$

$\tan^2 x + 2\tan x + 3 = 0$

$(\tan x - 3)(\tan x + 1) = 0$

$\tan x = 3 \quad \boxed{x = 71.56} \qquad \tan x = -1 \quad \boxed{x = 135°}$

$\boxed{x = -45 \text{ OR } 315°}$

17. $3\sec^2 x - 4 = 0$

$\frac{3\sec^2 x}{3} = \frac{4}{3}$

$\sqrt{\sec^2 x} = \sqrt{\frac{4}{3}}$

$\sec x = \pm\frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$

$\cos x = \pm\frac{\sqrt{3}}{2}$

$\boxed{x = \frac{\pi}{6}, \frac{11\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}}$

18. $3\cos x + 3 = 2\sin^2 x$

$-3\cos x - 3$

$2\sin^2 x - 3\cos x - 3 = 0$

$2(1 - \cos^2 x) - 3\cos x - 3 = 0$

$2 - 2\cos^2 x - 3\cos x - 3 = 0$

$-2\cos^2 x - 3\cos x - 1 = 0$

$2\cos^2 x + 3\cos x + 1 = 0$

$2\cos^2 x + 2\cos x + \cos x + 1 = 0$

$2\cos x(\cos x + 1) + 1(\cos x + 1) = 0$

$(2\cos x + 1)(\cos x + 1) = 0$

$\boxed{x = \frac{2\pi}{3}, \frac{4\pi}{3}, \pi}$

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