

ANSWER KEY (UNIT FOUR): 4.1 BASIC OPERATIONS

1. $\begin{bmatrix} 3 & -8 & 7 & 6 \\ 5 & 12 & 0 & -2 \end{bmatrix}$ DIMENSION: 2×4

2. $\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$ $a_{3,1} = 5$

3. $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} 6 & 8 \\ 7 & 9 \end{bmatrix} = \begin{bmatrix} 7 & 10 \\ 10 & 13 \end{bmatrix}$

4. $\begin{bmatrix} 7 & 12 \\ 6 & 3 \end{bmatrix} - \begin{bmatrix} 4 & 10 \\ 12 & 4 \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ -6 & -1 \end{bmatrix}$

5. $\begin{bmatrix} x & 14 \\ 1 & 3 \end{bmatrix} + \begin{bmatrix} 6 & 2 \\ -4 & y \end{bmatrix} = \begin{bmatrix} 10 & 16 \\ -3 & 6 \end{bmatrix}$

$$\begin{array}{r} x+6=10 \quad 3+y=6 \\ -6 \quad -6 \quad -3 \quad -3 \\ x=4 \quad y=3 \end{array}$$

6. $\begin{bmatrix} 4 & 3 \\ 6 & 2 \\ 7 & -1 \end{bmatrix} \cdot \begin{bmatrix} 2 & 4 \\ 8 & -1 \end{bmatrix} = \begin{bmatrix} 4(2)+3(8) & 4(4)+3(-1) \\ 6(2)+2(8) & 6(4)+2(-1) \\ 7(2)+(-1)(8) & 7(4)+(-1)(-1) \end{bmatrix}$

$$= \begin{bmatrix} 32 & 13 \\ 28 & 22 \\ 6 & 29 \end{bmatrix}$$

7. $\begin{bmatrix} 3 & 4x \\ 0 & 5 \end{bmatrix} \begin{bmatrix} 4y & -x \\ 0 & 5 \end{bmatrix} = \begin{array}{l} 3(4y)+4x(0) \quad 3(-x)+4x(5) \\ 12y \quad -3x+20x \\ \quad \quad \quad \quad \quad 17x \end{array}$

$$\begin{bmatrix} 12y & 17x \end{bmatrix}$$

4.2 Applications of Basic Operations

i) $20(15) + 35(15) + 15(15) + 15(20) + 42(20) + 24(20) + 18(25) + 38(25) + 22(25)$

$$\begin{array}{c|c|c} 300 + 525 + 225 & 300 + 840 + 480 & 450 + 950 + 550 \\ \hline FF = 1050 & C = 1620 & MJ = 1950 \end{array}$$

total: \$4620

ii) $6x + 3y + z = ?$ $57.3 = 26.2 + 21 + 10.1$
 $3x + 6y + z = ?$ $99.3 = 40.2 + 44.8 + 14.3$
 $3x + 1y + 6z = ?$ $218.2 = 71.9 + 63.5 + 82.8$

$$\begin{bmatrix} 6 & 3 & 1 \\ 3 & 6 & 1 \\ 3 & 1 & 6 \end{bmatrix} \times \begin{bmatrix} 57.3 \\ 99.3 \\ 218.2 \end{bmatrix}$$

$$3 \cdot 3 \quad 3 \cdot 1$$

$$\begin{bmatrix} 859.9 \\ 985.9 \\ 1580.4 \end{bmatrix}$$

$$\begin{bmatrix} 6(57.3) + 3(99.3) + 218.2 \\ 3(57.3) + 6(99.3) + 218.2 \\ 3(57.3) + 99.3 + 6(218.2) \end{bmatrix}$$

- total protein = 859.9 (g/cup)
- total carbs = 985.9 (g/cup)
- total fat = 1580.4 (g/cup)

ANSWER KEY (UNIT FOUR) = 4.3 INVERSE MATRICES

12.
$$\begin{pmatrix} 0 & 2 & 2 \\ 1 & 1 & 1 \\ 0 & 1 & 2 \end{pmatrix} = \begin{pmatrix} -.5 & 1 & 0 \\ 1 & 0 & -1 \\ -.5 & 0 & 1 \end{pmatrix}$$

13.
$$\begin{pmatrix} 8 & 12 \\ 2 & 3 \end{pmatrix} \quad \begin{matrix} 8 \cdot 3 - 12 \cdot 2 \\ 24 - 24 = 0 \end{matrix} \quad \text{NO INVERSE}$$

14.
$$\begin{pmatrix} 10 & 4 \\ 7 & 3 \end{pmatrix} \quad \begin{matrix} 10 \cdot 3 - 7 \cdot 4 \\ 30 - 28 = 2 \end{matrix} \quad \begin{pmatrix} 1.5 & -2 \\ -3.5 & 5 \end{pmatrix}$$

15.
$$\begin{pmatrix} 8 & 0 \\ 12 & 4 \end{pmatrix} = \begin{pmatrix} .125 & 0 \\ .375 & .25 \end{pmatrix} \quad \text{YES}$$

4.4 - Solving Systems w/3 Variables

16.)

$$\begin{aligned} 20x + 5y + 5z &= 25 \\ 15x + 15y - 10z &= 110 \\ 5x - 10y - 5z &= 15 \end{aligned}$$

$$\begin{array}{r} 20x + 5y + 5z = 25 \\ -20x + 40y + 20z = -60 \\ \hline 45y + 25z = -35 \\ -45y - 5z = -65 \\ \hline 20z = -100 \\ \frac{20}{20} \quad \frac{-100}{20} \\ \hline \boxed{z = -5} \end{array}$$

$$\begin{array}{r} 15x + 15y - 10z = 110 \\ -15x + 30y + 15z = -45 \\ \hline -1(45y + 5z = 65) \end{array}$$

$$\begin{array}{r} 15x + 15y - 10(-5) = 110 \\ -15x + 30y + 15(-5) = -45 \\ \hline 5x + 15y + 50 = 110 \\ -15x + 30y - 75 = -45 \\ \hline 45y - 25 = 65 \\ +25 \quad +25 \\ \hline 45y = 90 \\ \frac{45}{45} \quad \frac{90}{45} \\ \hline \boxed{y = 2} \end{array}$$

$$\begin{aligned} 20x + 5(2) + 5(-5) &= 25 \\ 20x + 10 - 25 &= 25 \\ 20x - 15 &= 25 \\ +15 \quad +15 \end{aligned}$$

$$\begin{array}{r} 20x = 40 \\ \frac{20}{20} \quad \frac{40}{20} \\ \hline \boxed{x = 2} \end{array}$$

$$\boxed{\begin{aligned} x &= 2 \\ y &= 2 \\ z &= -5 \end{aligned}}$$

17.)

$$\begin{aligned} -4y - 12z &= 8 \\ 2(2y + 8z) &= -10 \\ 2x + 4y + 14z &= -2 \end{aligned}$$

$$\begin{array}{r} -4y - 12z = 8 \\ 4y + 16z = -20 \\ \hline 4z = -12 \\ \frac{4}{4} \quad \frac{-12}{4} \\ \hline \boxed{z = -3} \end{array}$$

$$\begin{array}{r} -4y - 12(-3) = 8 \\ -4y + 36 = 8 \\ -36 \quad -36 \\ \hline -4y = -28 \\ \frac{-4}{-4} \quad \frac{-28}{-4} \\ \hline \boxed{y = 7} \end{array}$$

$$\begin{array}{r} 2x + 4(7) + 14(-3) = -2 \\ 2x + 28 - 42 \\ 2x - 14 = -2 \\ +14 \quad +14 \\ \hline 2x = 12 \\ \frac{2}{2} \quad \frac{12}{2} \\ \hline \boxed{x = 6} \end{array}$$

$$\boxed{\begin{aligned} x &= 6 \\ y &= 7 \\ z &= -3 \end{aligned}}$$

4.5 - Using Matrices to Solve Systems Answer Key.

18)
$$\begin{cases} 4y - z = 8x + 3 \\ x + 12z = 2y - 3 \\ 3y - 1 = 4x + 10z \end{cases}$$

$$\begin{aligned} 4y - z - 8x &= 3 \\ -2y + 12z + x &= -3 \\ 3y - 10z - 4x &= 1 \end{aligned}$$

$$\begin{bmatrix} 4 & -1 & -8 \\ -2 & 12 & 1 \\ 3 & -10 & -4 \end{bmatrix} \begin{bmatrix} y \\ z \\ x \end{bmatrix} = \begin{bmatrix} 3 \\ -3 \\ 1 \end{bmatrix}$$

$$X = A^{-1} \cdot B$$

calculator error

dependent = infinite solutions.

19) $x =$ amount in low risk
 $y =$ amount in high risk

$$\begin{aligned} x + y &= \$10,000 \\ .07x + .12y &= \$1,200 \end{aligned}$$

$$\begin{bmatrix} 1 & 1 \\ .07 & .12 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 10000 \\ 1200 \end{bmatrix}$$

$$X = A^{-1} \cdot B$$

$$\begin{aligned} x &\approx \$14,400 \\ y &\approx \$1,000 \end{aligned}$$

4.6 Augmented Matrices Answer Key

20)

$$-1x + 2y + 2z = 15$$

$$-x + 2y + 2z = 15$$

$$x + y - 2 = z \rightarrow x + y - z = 2$$

$$x + y - z = 2$$

$$3y - z - 18 = -2x$$

$$2x + 3y - z = 18$$

$$\text{rref} \left(\begin{array}{ccc|c} 1 & 2 & 2 & 15 \\ 1 & 1 & -1 & 2 \\ 2 & 3 & -1 & 18 \end{array} \right)$$

$$= \left(\begin{array}{ccc|c} 1 & 0 & 0 & -13 \\ 0 & 1 & 0 & 14.5 \\ 0 & 0 & 1 & -.5 \end{array} \right)$$

one solution / independent

$$x = -13 \quad y = 14.5 \quad z = -.5$$

21)

$$y = -3x + 3z + 13$$

$$3x + y - 3z = 13$$

$$-3x + 7z - 7y = 6$$

$$\rightarrow -3x - 7y + 7z = 6$$

$$-2x + 2z - 6y = 30$$

$$-2x - 6y + 2z = 30$$

$$\text{rref} \left(\begin{array}{ccc|c} 3 & 1 & -3 & 13 \\ -3 & -7 & 7 & 6 \\ -2 & -6 & 2 & 30 \end{array} \right)$$

$$= \left(\begin{array}{ccc|c} 1 & 0 & 0 & .625 \\ 0 & 1 & 0 & -7.25 \\ 0 & 0 & 1 & -6.125 \end{array} \right)$$

one solution / many solutions

$$x = .625 \quad y = -7.25 \quad z = -6.125$$