ICM Unit 5 Review (Functions & Limits) Name Key Date ________

1. Find the limit algebraically.

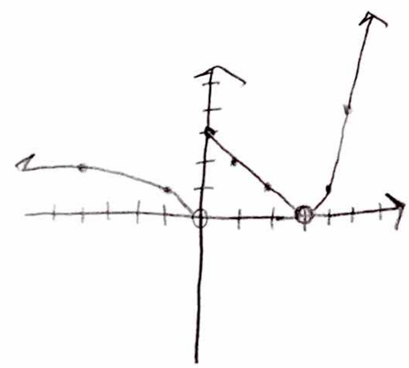
$$\lim_{x\to 0}\frac{(3-x)^2}{(x-3)} = \frac{(3-0)^2}{0-3} = \frac{3^2}{-3} = \frac{9}{-3} = \boxed{-3}$$

2. If y = 7 is a horizontal asymptote of a rational function f, then which of the following is true

a) $\lim_{x\to 7} f(x) = \infty$ b) $\lim_{x\to\infty} f(x) = 7$ c) $\lim_{x\to 0} f(x) = 7$

d) $\lim_{x\to 7} f(x) = 0$ e) $\lim_{x\to -\infty} f(x) = -7$

3. Let $f(x) = \begin{cases} \sqrt{-x}, & x<0 \\ 3-x, & 0\le x<3 \\ (x-3)^2, & x>3 \end{cases}$ Find each of the following:



a) $\lim_{x\to 0^+} f(x) = 3$ b) $\lim_{x\to 0^-} f(x) = 0$

c) $\lim_{x\to 0} f(x) =$ DNE d) $\lim_{x\to 3^-} f(x) = 0$

e) $\lim_{x\to 3^+} f(x) = 0$ f) $\lim_{x\to 3} f(x) = 0$

4. Let the function f be defined by $f(x) = \begin{cases} \sqrt{x+1}, & 0\le x\le 3 \\ 5-x, & 3<x\le 5 \end{cases}$

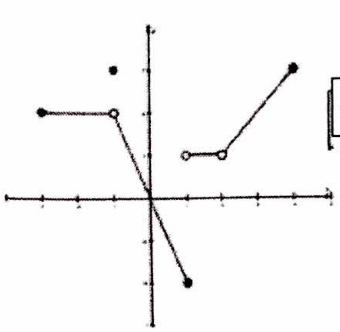
$\sqrt{3+1} = 2$

$5-3 = 2$

Is f(x) continuous at x = 3? Why or why not?

Yes. The two parts of the piecewise function meet at (3, 2).

5. Use the graph of f(x) to find each limit.



a) $\lim_{x\to 1^+} f(x)$	b) $\lim_{x\to 1^-} f(x)$	c) $\lim_{x\to 1} f(x)$	d) $\lim_{x\to -1} f(x)$	e) $\lim_{x\to 2} f(x)$
1	-2	DNE	2	2

Find the indicated limit. Which method is most appropriate: Direct Substitution, Numerical, Analytic or Graphical?

Direct Subst. 6. $\lim_{x\to -3}(3x+2)$ $\quad 3(-3)+2 = \boxed{-7}$

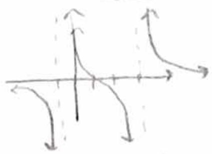
Direct Subst. 7. $\lim_{x\to -1}\frac{x^3-1}{x-1}$ $\quad \frac{(-1)^3-1}{-1-1} = \frac{-2}{-2} = \boxed{1}$

Algebraically 8. $\lim_{x\to -1}\frac{2x^2-x-3}{x+1}$ $= \lim_{x\to -1}\frac{(2x-3)\cancel{(x+1)}}{\cancel{x+1}} = \lim_{x\to -1}(2x-3) = 2(-1)-3 = \boxed{-5}$

Graphically 9. $\lim_{x\to 0^-}\frac{x+1}{x}$

$\boxed{-\infty}$

Graphically 10. $\lim_{x\to 3^+}\frac{x}{x^2-2x-3}$



$\boxed{\infty}$

Algebraically 11. $\lim_{h\to 0}\frac{(x+h)^3-x^3}{h}$ $= \lim_{h\to 0}\frac{\cancel{x^3}+3x^2h+3xh^2+h^3-\cancel{x^3}}{h} = \lim_{h\to 0} 3x^2+3xh+h^2 = \boxed{3x^2}$

Numerically 12. $\lim_{x\to\infty}\frac{8x^3-5x}{x^2-3x}$ no horizontal asymptote. graph doesn't show all. } In the table, as x increases, y increases so... $\boxed{\infty}$

13. $\lim_{x\to\infty}\left(\frac{6x^3-5x}{x^2+4x^3}\right)$ Horiz. Asymp: $y=6$ so $\boxed{6}$

14. $\lim_{x\to\infty}\frac{x^2+x^4}{x^2+x^6}$ Horiz Asymp: $y=0$ so $\boxed{0}$

15. Evaluate each when $f(x) = x^2 - 2x - 3$.

a) $f(x+4)$

$(x+4)^2-2(x+4)-3$

$x^2+8x+16-2x-8-3$

$\boxed{x^2+6x+5}$

b) $\frac{f(x+h)-f(x)}{h}$ $= \frac{(x+h)^2-2(x+h)-3-(x^2-2x-3)}{h}$

$= \frac{\cancel{x^2}+2xh+h^2-\cancel{2x}-2h-\cancel{3}-\cancel{x^2}+\cancel{2x}+\cancel{3}}{h}$

$= \boxed{2x+h-2}$

16. Find the average rate of change of $f(x) = x^3 - 2x + 1$ from -3 to -2.

$(-3,-20)$
$(-2,-3)$

$\frac{-3+20}{-2+3} = \frac{17}{1} = \boxed{17}$

17. For $f(x) = x^2 - 3$, write the equation of the secant line contain points (-2, g(-2)) and (1, g(1)).

$(-2,1)$ $\quad (1,-2)$

$m = \frac{-2-1}{1--2} = \frac{-3}{3} = -1$

$y = mx+b$
$1 = -1(-2)+b$
$1 = 2+b$
$-1 = b$

$\boxed{y=-x-1}$

State the domain for each of the following. Write answers in interval notation.

12. $y = \dfrac{x+2}{x-11}$ $\quad x \neq 11$ $\quad \boxed{(-\infty, 11) \cup (11, \infty)}$

13. $y = \dfrac{2}{10x^2 - 33x - 7}$ $\quad (5x+1)(2x-7)$ $\quad x \neq -\tfrac{1}{2} \quad x \neq \tfrac{7}{2}$ $\quad \boxed{(-\infty, -\tfrac{1}{2}) \cup (-\tfrac{1}{2}, \tfrac{7}{2}) \cup (\tfrac{7}{2}, \infty)}$

14. $y = 7^x$ $\quad \boxed{(-\infty, \infty)}$

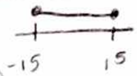
15. $y = \dfrac{\sqrt{4x-7}}{\sqrt{12-x}}$ $\quad 4x - 7 \geq 0 \quad x \geq \tfrac{7}{4}$ $\quad 12 - x > 0 \quad 12 > x \quad x < 12$ $\quad \boxed{[\tfrac{7}{4}, 12)}$

16. $f(t) = \dfrac{1}{\sqrt{t^2 - 121}}$ $\quad t^2 - 121 > 0$

(16) $\boxed{(-\infty, -11) \cup (11, \infty)}$

17. $y = \sqrt{225 - x^2}$

(17) $225 - x^2 \geq 0$



$\boxed{[-15, 15]}$

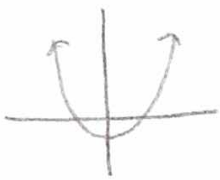
18. $f(x) = \dfrac{\sqrt{x-4}}{x-7}$

(18) $x - 4 \geq 0 \quad x \geq 4 \quad x \neq 7$ $\quad \boxed{[4, 7) \cup (7, \infty)}$

19. $y = 3x^2 + 2$ $\quad \boxed{(-\infty, \infty)}$

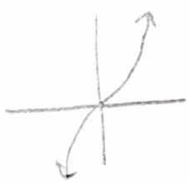
<u>Sketch the graph</u> and state the range of each. Write answers in interval notation.

20. $f(x) = x^2 - 1$



$[-1, \infty)$

21. $f(x) = x^3$

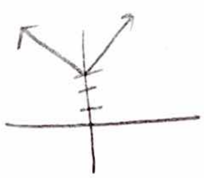


$(-\infty, \infty)$

22. $f(x) = \sqrt{x-3}$

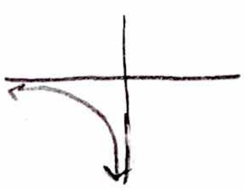
$[0, \infty)$

23. $f(x) = |x| + 3$



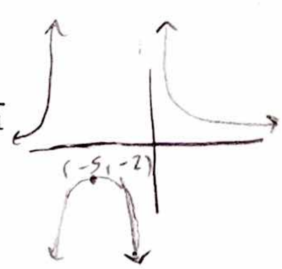
$[3, \infty)$

24. $h(x) = -9(2^x)$



$(-\infty, 0)$

25. $f(x) = \dfrac{8}{x^2 + 10x + 21}$



$(-\infty, -2) \cup (0, \infty)$

26. Dana's cell phone plan costs $42 per month for 1000 minutes (regardless of the time of day). For each additional minute, she is charged 5¢.

a) Write a function for the cost, C, of Dana's cell phone plan in terms of minutes, m.

$$C = \begin{cases} 42 & \text{if } m \leq 1000 \\ 42 + .05(m - 1000) & \text{if } m > 1000 \end{cases}$$

b) If she spends $60 in March for her cell phone plan, how many minutes will she use?

$60 = 42 + .05(m - 1000)$

$18 = .05(m - 1000)$

$300 = m - 1000$

$m = 1300$

1300 minutes

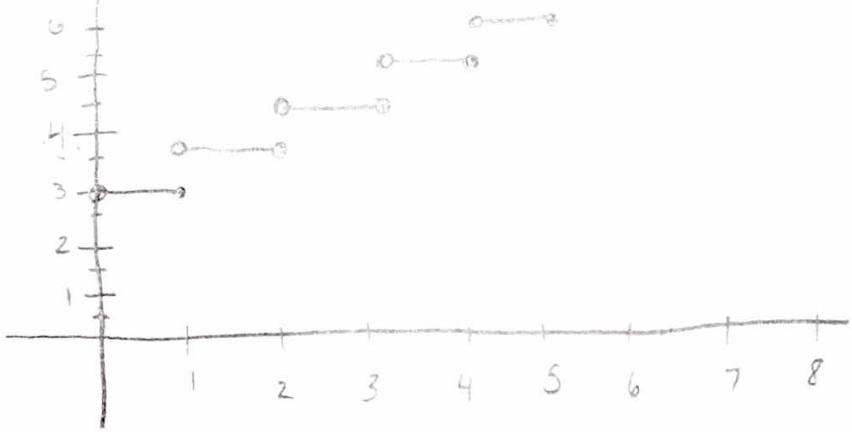
27. The post office charges $3.00 to mail a package weighing up to (and including) 1 pound and $0.75 for each additional pound or portion of a pound.

a) Use a step function to write an equation for the total cost C for sending a package weighing x pounds.

$C = 0.75\lceil x - 1 \rceil + 3$

b) Find the cost of mailing a package that costs 3.6 pounds. $3 + \lceil 2.6 \rceil .75 = \$5.25$

c) Graph the function on the domain $0 \leq x \leq 8$.



I ran out of vertical space :(

You get the picture :)