

Unit 3 Review

* = No Calculator!

*A radical expression can be written as a rational exponent, and vice versa.

$$\sqrt[3]{x} = x^{1/3} \quad y^{\frac{4}{5}} = \sqrt[5]{y^4}$$

1. *Simplify the following.

a) $\sqrt[3]{x^2yz}$

b) $\sqrt[4]{x^5y^{10}z} = \boxed{xy^2\sqrt{xy^2z}}$

c) $\sqrt[2]{75a^{11}b^4c^7}$

$$5a^5b^2c^3\sqrt{3ac}$$

2-(-3/4) Simplest form

2 3/4

2 3/4

2 1/4

X

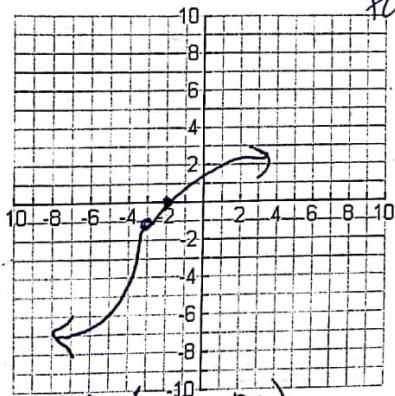
d) $\frac{2x^2y^{1/2}}{4x^{-3/4}y^{1/3}} = \boxed{\frac{x^{11/4}y^{1/6}}{2}}$

e) $(a^4b^6c^{-1/2})^{1/4} = \boxed{\frac{ab^{3/2}}{c^{1/8}}}$

2. *Graph. State the transformations in order. State the Domain and Range.

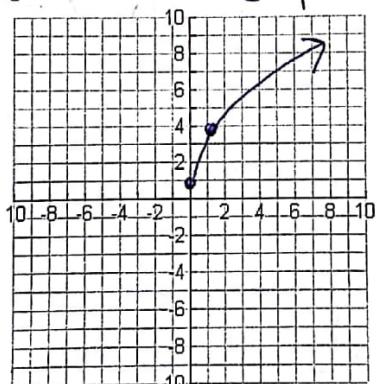
a) $y = \sqrt[3]{x+3} - 1$ (-3, -1)

Can use calculator for cube root



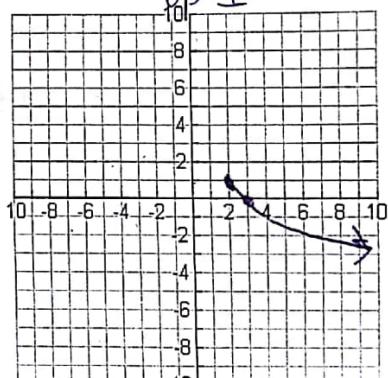
D: $(-\infty, \infty)$
R: $(-\infty, \infty)$

b) $y = 3\sqrt{x+1}$
Up 1
Slope = 3



D: $[0, \infty)$
R: $[0, \infty)$

c) $y = -\sqrt{x-2} + 1$
-reflect over x-axis
-right 2
-up 1



D: $[2, \infty)$
R: $(-\infty, 1]$

3. *Working backwards: Writing the equation when given a translation.

- a) The parent function
- $y = 1/x$
- is translated 2 units to the left and one unit down.

$$y = \frac{1}{x+2} - 1$$

- b) The parent function
- $y = \sqrt{x}$
- is translated 3 units to the right and reflected across the x-axis.

$$y = -\sqrt{x-3}$$

- c) The parent function
- $y = \sqrt{x}$
- is compressed vertically by a factor of
- $\frac{1}{2}$
- and 2 units up.

$$y = \frac{1}{2}\sqrt{x} + 2$$

(*) make sure to check!

4. Solve each radical equation. Show all work for full credit! Be sure to check for extraneous solutions.

a) $\sqrt[3]{x+1} = 4$

$$\begin{array}{rcl} x+1 & = & 64 \\ -1 & & -1 \\ \boxed{x=63} & & \end{array}$$

$$\begin{array}{rcl} 3\sqrt{63+1} & = & 4 \\ 3\sqrt{64} & = & 4 \\ 4 & = & 4 \checkmark \end{array}$$

b) $5\sqrt{x+7} = 25$

$$\begin{array}{rcl} 5 & & 5 \\ \sqrt{x+7} & = & 5 \\ (\sqrt{x+7})^2 & = & 5^2 \\ x+7 & = & 25 \\ -7 & & -7 \\ \boxed{x=18} & & \end{array}$$

$\boxed{x=18}$

Check: $5\sqrt{18+7} = 25$

$$\begin{array}{rcl} 5\sqrt{25} & = & 25 \\ 5 \cdot 5 & = & 25 \\ 25 & = & 25 \checkmark \end{array}$$

c) $(\sqrt{3x-1})^2 = (\sqrt{2x+4})^2$ Check:

$$\begin{array}{rcl} 3x-1 & = & 2x+4 \\ -2x & & -2x \\ x-1 & = & 4 \\ +1 & & +1 \\ \boxed{x=5} & & \end{array}$$

$$\begin{array}{rcl} \sqrt{3(5)-1} & = & \sqrt{2(5)+4} \\ \sqrt{14} & = & \sqrt{14} \checkmark \end{array}$$

d) $10 - 2\sqrt{3x-1} = -14$

$$\begin{array}{rcl} 10 & & -14 \\ -2\sqrt{3x-1} & = & -24 \\ -2 & & -2 \\ (\sqrt{3x-1})^2 & = & (12)^2 \\ 3x-1 & = & 144 \\ \boxed{x=48.3} & & \end{array}$$

$\boxed{x=48.3}$

Check:

e) $(2x+5)^3 = 16^{3/2}$

$$\begin{array}{rcl} 2x+5 & = & 64 \\ -5 & & -5 \\ 2x & = & 59 \\ \cancel{2} & & \cancel{2} \\ \boxed{x=29.5} & & \end{array}$$

$\boxed{x=29.5}$

Check:

f) $(4x+8)^2 = (2x)^2$

$$\begin{array}{rcl} 4x+8 & = & 4x^2 \\ -4x-8 & & -4x-8 \\ 0 & = & 4x^2 - 4x - 8 \\ 0 & = & 4(x^2 - x - 2) \\ 0 & = & 4(x-2)(x+1) \end{array}$$

Check:

$(4(2)+8)^{1/2} = (2(2))^{1/2}$

$$\begin{array}{rcl} 16^{1/2} & = & 4 \\ 0 & = & 4x^2 - 4x - 8 \\ 0 & = & 4(x^2 - x - 2) \\ 0 & = & 4(x-2)(x+1) \end{array}$$

$4=4 \checkmark$

$16^{1/2} = 4$

$4(-1)+8)^{1/2} = 2(-1)$

$4^{1/2} = 2$

$2=2 \checkmark$

g) $2 = -x + \sqrt{2x+3}$

$$\begin{array}{rcl} +x & & +x \\ (x+2)^2 & = & (\sqrt{2x+3})^2 \\ (x+2)(x+2) & = & 2x+3 \\ x^2 + 4x + 4 & = & 2x+3 \end{array}$$

$\boxed{x=-1}$

Check:

5. Write an equation of variation to model the following. Then find the missing information.

- a. The time to complete a project varies inversely with the number of employees. If 3 people can complete the project in 7 days, how long will it take 5 people?

$$y = \frac{k}{x} \rightarrow 7 = \frac{k}{3} \quad \underline{k=21} \quad y = \frac{21}{5} = \boxed{4.2}$$

- b. The volume V of a gas kept at a constant temperature varies inversely as the pressure p. If the pressure is 24 pounds per square inch, the volume is 15 cubic feet. What will be the volume when the pressure is 30 pounds per square inch?

$$V = \frac{k}{p} \quad \underline{V = \frac{k}{24}} \quad \underline{k=360} \quad V = \frac{360}{30} = \boxed{12}$$

- c. The distance a body falls from rest varies directly as the square of the time it falls (ignoring air resistance). If a ball falls 144 feet in three seconds, how far will the ball fall in seven seconds?

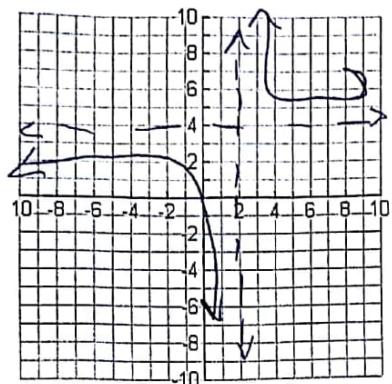
$$\begin{array}{l} y = Kx^2 \rightarrow \\ d = Kt^2 \rightarrow 144 = K(3)^2 \\ \boxed{144 = 9K} \end{array}$$

$\underline{K=16}$

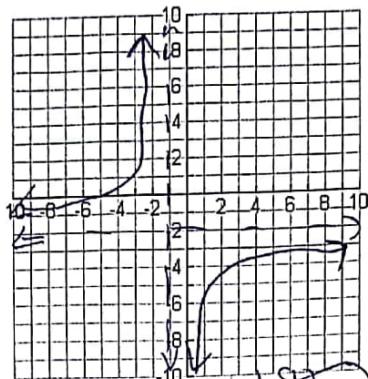
$$\begin{array}{l} d = 7^2 \cdot 16 \\ d = 49 \cdot 16 \\ \boxed{d = 78.4 \text{ ft.}} \end{array}$$

6. * Graph the following rational functions. State all asymptotes.

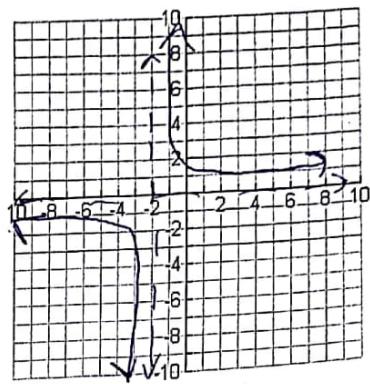
a. $y = \frac{1}{x-2} + 4$ VA: $x=2$
HA: $y=4$



b. $y = \frac{-1}{x+1} - 2$ VA: $x=-1$
HA: $y=-2$



c. $y = \frac{3}{x+2}$ VA: $x=-2$
HA: $y=0$



7. Solve the following systems algebraically.

a. $\begin{cases} y = \frac{4}{x} + 1 \\ y = 2 - x \end{cases}$ $x - x^2 = 4$
 $-x + x^2 - x + x^2$

Imaginary Solutions
No Solutions

b. $y = x + 1$ and $y = 72/x$
 $x(x+1) = \frac{72}{x}$

$x^2 + x = 72$
 $-72 -72$

$x = 8$ $x = -9$
 $(8, 9)$ $(-9, -8)$

Solutions

$2-x = \frac{4}{x} + 1$
 -1
 $x(1-x) = \left(\frac{4}{x}\right)x$

$0 = x^2 - x + 4$
 $-b \pm \sqrt{b^2 - 4ac}$
 $-\frac{(-1) \pm \sqrt{(-1)^2 - 4(1)(4)}}{2(1)}$
 $1 \pm \sqrt{1-16}$

$x^2 + x - 72 = 0$
 $(x-8)(x+9) = 0$

8. The function $S = \pi \sqrt{\frac{9.8L}{7}}$, can predict the maximum speed a person can run if S is the speed in meters per second and L is the length of the person's leg in meters.

a) How long is a person's leg if his maximum speed is 3.32 meters per second?

$$\frac{3.32}{\pi} = \sqrt{\frac{9.8L}{7}} \quad \left(\frac{3.32}{\pi}\right)^2 = \left(\sqrt{\frac{9.8L}{7}}\right)^2 \quad \frac{1.11}{1.4776} = \frac{1.11}{1.4776} = 1.4$$

Ex. 8m

b) According to this function, would a person run faster or slower if their legs were longer. Show your work to justify your answer.

9. In the formula $c = \sqrt{h^2 + r^2}$, c is the slant height of a cone, h is the height of the cone, and r is the radius of the base. Find the height of the cone if the slant height is 8 and the radius is 3. Round your answer to the nearest hundredth.

$$8 = \sqrt{h^2 + 3^2}$$

$$(8)^2 = (\sqrt{h^2 + 9})^2$$

$$64 = h^2 + 9$$

$$\sqrt{55} = h$$

$$7.42 \approx h$$